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REPORT OF THE NORTH SEA YOUNG HERRING WORKING GROUP

by

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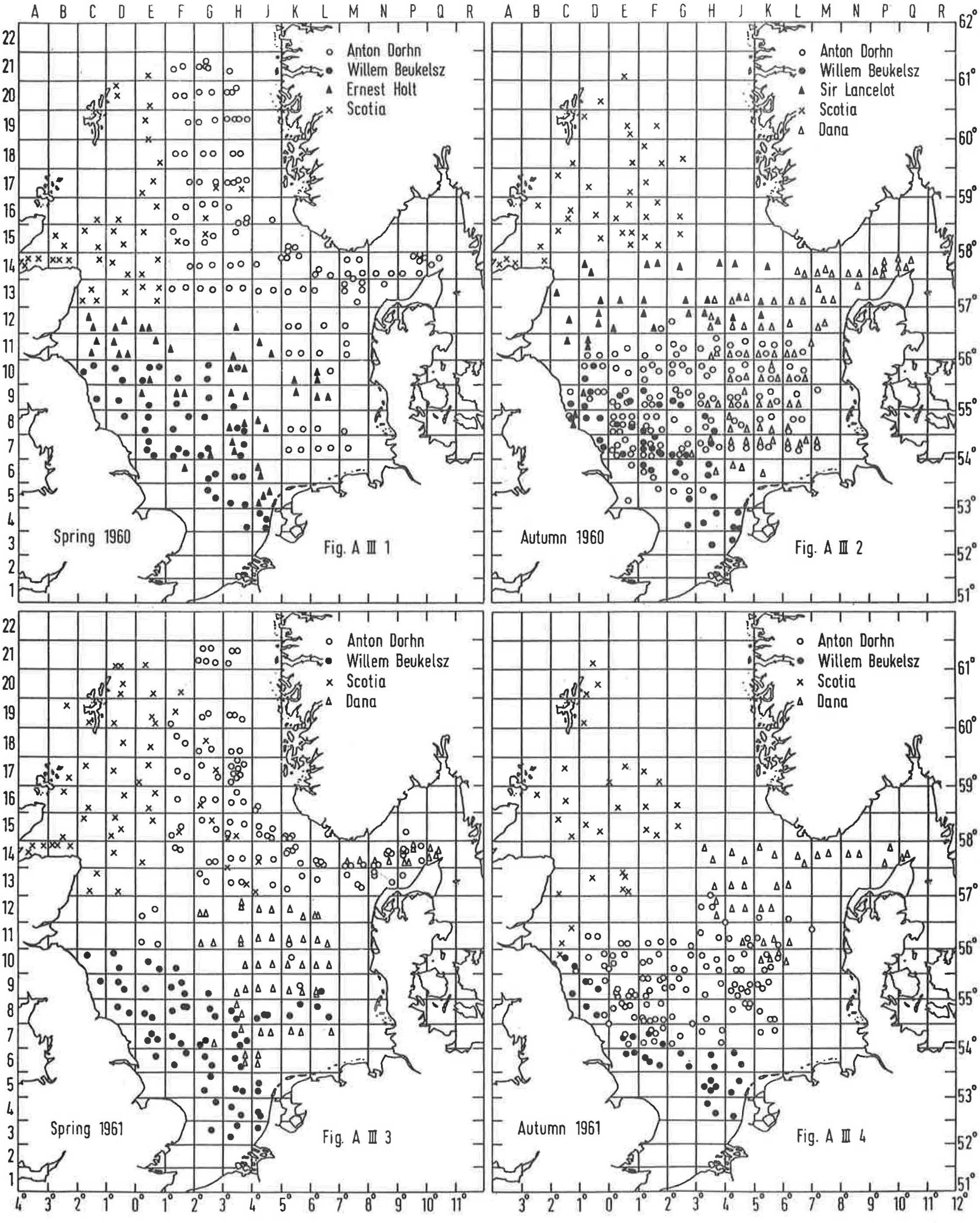


Figures A III 1-4 Distribution of the hauls made by the research vessels.

A INTRODUCTION

A I History of the project

At the meeting of the Herring Committee in October 1959 a recommendation was passed that "in furthering the co-ordination of, and progress in, herring larval and recruitment studies, conjoint programmes of work should be drawn up by herring scientists of Germany, Denmark, Netherlands, England and Scotland, and that, as part of this programme, interested countries should undertake conjoint research vessel trawling surveys in spring and autumn respectively". Arising from this recommendation an ad hoc working group, consisting of the representatives of these five countries, met in Copenhagen and drew up plans for implementing it.

It was agreed that the objectives of the surveys should be to identify the main centres of abundance of pre-recruit herring in the North Sea, and to determine their racial characteristics in relation to those of the adult stocks. All five countries agreed to participate in achieving these objectives by making surveys in March-April and in September 1960, covering the whole of the North Sea and outer regions of the Skagerak from north of Shetland to the Southern Bight.

For the first cruise, in March 1960, the research ships allocated to this project were "Dana", "Willem Beukelsz", "Ernest Holt", "Anton Dohrn" and "Scotia". "Dana" broke down immediately prior to the beginning of the survey and was unable to participate. However, Danish scientists mounted an intensive sampling and statistics collection from the Danish industrial fishery, in order to sample as completely as possible the area which should have been covered by "Dana". The second cruise was carried out in September 1960 by the same ships with the exception of "Ernest Holt", which was replaced by "Sir Lancelot".

At the meeting of the Herring Committee in Moscow in October 1960 it was agreed that these surveys should be continued in 1961 along the same lines as in 1960, because one year's data were considered insufficient to give conclusive results.

At the meeting of the Herring Committee in October 1961 it was agreed that these young herring trawling surveys should be discontinued in 1962, pending the full analysis of the data collected. It was also decided that the help of the Comparative Fishing Committee should be sought in the estimation of the relative fishing powers of the ships taking part in these surveys.

At the 1962 meeting of the Herring Committee the Chairman announced that the exchange of data collected on these surveys had been completed. It was agreed that representatives of the countries concerned should meet for two days at the time of the meeting of the North Sea Working Group in IJmuiden, to make a first appraisal of the data and to consider future plans. This group met in IJmuiden in March 1963 and a report of their deliberations was presented to the Herring Committee in October 1963 (Anon 1963). The main conclusion of this meeting was that the extent and complexity of the material made automatic processing of the data essential. An offer by the English laboratory to process these data by computer was accepted and a working group consisting of J. J. Zijlstra (Convener), A. C. Burd, K. Popp Madsen, A. Schumacher and A. Saville was nominated to plan and carry out the further processing and analysis of the data. The meeting of the Herring Committee in Madrid in 1963 confirmed the nominations to this working group and recommended that it should meet in London as soon as possible to set up the computer program. It was also recommended that this group should be joined by K. P. Andersen in view of his special knowledge of the techniques of discriminant function analysis.

The group met in London in November 1963 and made plans for the preparation of the data for the computer and for the way in which the discriminant function analysis should be carried out. It was reported to the Herring Committee in October 1964 that the computer processing was then in progress (Zijlstra 1964).

A further meeting of the working group took place in Copenhagen in June 1965 to discuss the preliminary results of the computer discriminant analysis. This gave results which were obviously untenable and the reasons for this were discussed at length. It was decided that the major factor responsible for the breakdown in the discriminant analysis probably lay in the discrepancies between the variances of the true l_1 values used for the Pure Stocks and the standardized l_1 values calculated for the recruit herring. At this meeting arrangements were also made for the preparation for publication of that part of the survey data which was not affected by the computer analysis. A report of this meeting of the working group was presented to the Herring Committee in October 1965 (Burd 1965).

The working group held a further meeting at Lowestoft in April 1966, where plans were laid for a new computer discriminant analysis, using standard l_1s as a Pure Stock character and utilizing all maturity stages of the young herring instead of restricting these to maturity stages I and II as in the first computer analysis.

The working group met again at Lowestoft in June 1967. At this meeting most of the final computer analysis was available for study, and final plans were made for the publication of all the data collected on the surveys in 1960 and 1961.

An editorial meeting of the working group took place in IJmuiden in April 1968, and a first draft of the report was presented at the 1968 annual meeting of ICES. A final editorial meeting was held in IJmuiden in March 1969.

A II Objectives

There has been a long time interval between the collection of the material during the cruises in 1960-61 and the publication of the results, although some of the material has been quoted in progress reports presented to the Herring Committee of ICES (Anon 1963, Zijlstra 1964, Burd 1965, Burd 1967).

One of the main causes of the delay between data-collection and publication has been the need for a better definition and an extension of the original objectives of the experiment, after completion of the surveys.

At the start of the exercise its objectives were formulated shortly as: (1) identifying the main centres of abundance of pre-recruit herring in the North Sea, and (2) determining the "racial" and other biological characteristics of the immature fish in relation to the characteristics of the adult stocks.

The first objective - identification of the main nursery areas of North Sea herring - could be achieved with little trouble after circulation of the data on catches, by simply plotting the catches of immature herring by the different vessels. Additional information on age and length, as shown in section B I, refines the picture without altering it to a large extent. The operation was thus fairly successful in determining the major nurseries of North Sea herring in the years 1960-61. It was also successful in attaining an additional aim of the exercise, i.e. determining the hydrographical situation in the North Sea at the time of the surveys. The results of this study have been published (ICES 1966). In the present paper an attempt is made to connect the distribution of immature fish with the hydrographical situation in each of the four surveys (section B III).

It has, however, been the second objective, i.e. relating the immatures to the stocks of adult herring, which has caused most trouble and a considerable delay in the analysis of the material. The Working Group which discussed the final analysis of the material in IJmuiden in 1963, and the authors of this paper, interpreted the second objective to be that the analysis should be aimed at elucidating the pattern of distribution and movements of the immatures of each of the three North Sea autumn-spawning herring stocks (Buchan, Bank and Downs) separately.

A first analysis of the data of the cruises suggested that the immatures of the three stocks occurred mixed together in all the nurseries (Anon 1963). The second objective could therefore only be attained by determining the proportion of the three stocks among the immatures, and this means distinguishing fish of different stock in the mixtures. North Sea herring research has as yet found no means of separating herring of different stocks individually. However, the North Sea Herring Working Group, which met subsequent to the meeting of the Working Group on the young herring in IJmuiden in 1963, tested a method which could in theory determine the mixing rates of the three stocks of North Sea herring on a group character basis. (See Report of the North Sea Herring Working Group 1963, Annex III, C.M. 1963, Herring Comm. Paper no. 71.) This method, discriminant function analysis, described in section D I, cannot answer the question of to which stock the individual fish belong, but does provide a means of determining the proportion of each stock in a mixture.

The authors of this paper accepted the method, but realized that in view of the extent of the material, amounting to data on some 35 000 herring, and the complexity of the calculations involved, a form of mechanized processing of the data was inevitable. The offer of the use of an English computer made such a procedure possible.

An understanding was reached with the North Sea Herring Working Group, which was highly interested in the method because of similar mixing problems in the adult North Sea herring, that the method would be applied to the young herring data on an experimental basis. Once a computer program was ready and working for the immature herring it was considered an easy matter to adapt the program to the mixing problems of the adult fish.

Adapting the data of the immature herring to computer analysis, and especially preparing a program, consumed much time, if only because the material was not collected with this treatment in mind. There is little doubt among the authors that, had the surveys been properly planned for this analysis, the results would have been more reliable and much time would have been saved.

In addition to the problems related to adapting the data to computer analysis, special difficulties were encountered which are thought to have affected the results of the experiment. The first of these concerns information obtained about the abundance of fish from the size of the catch. This problem includes such questions as the variance on a haul, the relative fishing power of the participating vessels, the availability of the herring in different areas, etc., and has a bearing on the relative importance of the different nurseries, on the distribution and abundance of the immature fish of the three stocks and last, but not least, on the relative size of the three North Sea herring stocks. In several sections of this paper this problem will arise, but in particular one section (B IV) is devoted to it.

Other problems were encountered in relation to the discriminant function analysis, which was carried out on the assumption that all the immature herring belonged to one of the three stocks. If the immature fish analysed contained representatives of other, unknown, stocks this would invalidate the analysis, to a degree depending on the proportion of such fish among the immatures.

Three characters were used in the discriminant function analyses. Little trouble was encountered with two characters, namely vertebral counts (VS) and keeled scale counts (K_2). The most important character for the analysis, l_1, which possesses by far the largest discriminant power, gave considerable trouble, as is discussed in section C II.

Finally, the possibility that the immatures showed segregation by length has been a constant concern, because a condition for the proper operation of discriminant function analysis is that all components of the three stocks are available in each nursery. Segregation by length, in which all the small fish of the three stocks are to be found in one part of the nursery area whereas all the large fish are in another area, would invalidate the analysis completely. In view of the distribution of the immatures in relation to length (see section B I), segregation by length seemed possible. Such segregation should, however, show up in the results of the analysis, and is discussed under section D II 2.

A III Surveys

The effort of the research vessels participating in the four cruises was distributed according to a plan which allotted a rather large unit area to each ship. However, due to trouble with ships and weather, the surveys did not always follow the programme, and ad hoc decisions had to be made at sea (by radio contact) between ships to fill in gaps in the area to be covered where one of the vessels failed to finish its work. In the original plan the areas allotted to the different vessels were supposed to overlap, with the intention of obtaining information on the relative fishing power of the ships. In practice, however, the vessels were fully occupied sampling the whole of the area, without bothering about an overlap.

The result of the cruises, in terms of fishing area and distribution of hauls, are shown in Figures A III 1-4, on which the mean position of each haul has been plotted, by vessels.

Table A III 1 gives a summary of these charts, and also the survey period of each of the vessels. From the charts and the table it can be seen that the coverage of the area was good in spring and autumn of 1960 and in the spring of 1961 (respectively 112, 110 and 128 squares fished), whereas it was rather poor in the autumn of 1961 (74 squares fished). In most squares two or more hauls were made, generally by only one vessel, but sometimes (autumn 1960) by two or even three vessels. The duration of the surveys was sometimes rather long, because ships had other commitments, which caused a bad synchronization of the surveys of the vessels. The spring 1960 survey lasted from 26 February to 12 April ($6\frac{1}{2}$ weeks); autumn 1960: from 26 August to 11 October ($6\frac{1}{2}$ weeks); spring 1961: from 14 February to 19 April (9 weeks); autumn 1961: from 3 September to 13 October (6 weeks). This long duration certainly affected the planned synoptic character of the experiment.

Table A III 1 Participation by research vessels in the surveys

Season and year	Ship	General area in North Sea	Period	Number of hauls
Spring 1960	"Anton Dohrn"	NE	18 Mar- 9 Apr	114
	"Ernest Holt"	Middle	19 Mar- 7 Apr	44
	"Scotia"	NW	26 Feb -12 Apr	42
	"Willem Beukelsz"	SW	7 Mar-24 Mar	47
				247
Autumn 1960	"Anton Dohrn"	SE + SW	15 Sep -11 Oct	128
	"Dana"	SE	26 Aug -17 Sep	78
	"Scotia"	NW	7 Sep -10 Oct	37
	"Sir Lancelot"	Middle	5 Sep -27 Sep	33
	"Willem Beukelsz"	SW	6 Sep -26 Sep	47
				323
Spring 1961	"Anton Dohrn"	NE	15 Mar-11 Apr	109
	"Dana"	SE	5 Apr -19 Apr	60
	"Scotia"	NW	14 Feb -19 Apr	53
	"Willem Beukelsz"	SW	6 Mar-12 Apr	64
				286
Autumn 1961	"Anton Dohrn"	SE + SW	13 Sep -13 Oct	113
	"Dana"	SE	3 Sep -21 Sep	35
	"Scotia"	NW	7 Sep -10 Oct	30
	"Willem Beukelsz"	SW	5 Sep -14 Sep	31
				209

The original plan included a proposal for echo-surveys. However, it was fairly evident from the results of the first cruises that in the area of high abundance of immature herring traces were rare, and catches of herring bore no relation whatsoever to echo-traces. Moreover, it was observed that good traces often did not coincide with herring catches, but with catches of other fish (whiting, cod, etc.), especially in spring. For this reason no attempt has been made to provide maps of the echo-surveys carried out during the cruises.

A IV Sampling

Sampling the herring stocks was done exclusively by trawl. The use of standard gear was considered at the initial planning stage but was decided against because the fishing characteristics of the participating vessels were so different that the use of the same gear would not eliminate differences in fishing power. In order to reduce between-survey variations in fishing power it was therefore decided that each ship should use its normal gear, preferably without alterations, throughout the four surveys.

The maximum mesh size in the cod-end was fixed at 25 mm, corresponding to a 50 per cent selection length for herring of about 10 cm. In trawls with larger cod-end meshes a fine-meshed cover or inner lining was used.

In order to diminish the influence of diurnal variations in catchability, all hauls were carried out in daylight. As far as possible, the duration of haul ($\frac{1}{2}$-1 hour) was kept constant within each ship's surveys. Surface and bottom temperatures and salinities were taken at each trawl station, so as to provide a synoptic picture of these hydrographic features to relate to the distribution of herring (see section B III).

The following treatment of the catch was adopted on all ships:

(i) For each haul, the catch of herring and other species was measured in baskets of specified capacity.

(ii) When the total catch of herring was one, or less than one, basket all herring were counted and then either all, or a sample, of them treated in detail.

(iii) When the herring catch exceeded one basket, one basket was selected at random and its contents counted, and the number raised to the total catch.

A random sample of at least 200-300 fish from the counted basket was measured and a random sub-sample of 100 fish from this sample was then selected for more detailed analysis.

In accordance with the ICES-ICNAF recommendation arrived at in Lisbon in 1957 it was decided to record all measurements to the nearest centimetre. Some countries preferred measurements being taken to the centimetre below, a difficulty that was overcome by taking bulk measurements to the half centimetre below. In that way length frequency data could be presented both in the national customary way and to the nearest centimetre. In fact, the young herring surveys stressed the need for greater uniformity of the methods used by herring research workers and led to the establishment of the Herring Working Group on Methods (Anon 1962).

Each fish selected for more detailed analysis was measured to the nearest millimetre, and sex and maturity were determined. Otoliths were collected for age reading and otolith typing, and scales were collected for l_1 determination. In most cases VS and K_2 were counted.

A V Comparability of characters between laboratories

The initial discussions on the data obtained by the surveys led to a number of questions concerning the comparability of characters measured by different laboratories. Some of the characters included in routine analysis of herring samples are estimated on a rather subjective basis, e.g. maturity and otolith types, while other characters like l_1, K_2 and VS, though based on direct measurements or countings, may show consistent differences due to differences in the methods used or in the definition of the character is question.

In order to test the comparability of characters based on scales and otoliths, six samples of scales and otoliths of North Sea herring were circulated amongst the Danish, Dutch, English, German and Scottish laboratories in 1961-62. Each laboratory did routine determinations of age, l_1, l_2 and otolith types. The analysis of this exercise is described in Appendix 1 and a short summary of the results is given below:

(i) Age

Age readings were done on both scales and otoliths. There was some indication of laboratory differences in the scale readings but not in otolith readings. It was, however, not possible to determine which of the two methods of age determinations is to be preferred.

(ii) l_1-l_2

The analysis revealed highly significant and consistent differences between the laboratories in l_1 and l_2 measurements. The largest difference between two laboratories was 4.13 mm in l_1, and 2.97 mm in l_2. This led incidentally to the discovery that the measuring apparatus in one of the laboratories suffered from a mechanical defect. The measuring error, as given by the variances, was of the order of 3 mm^2. This is probably an underestimate, since only the best scales were used, i.e. only those scales that all five laboratories included in their measurements.

(iii) Otolith typing

The data showed clear differences in the determination of otolith types, i.e. in the characterization of the first winter ring as being narrow, wide, or unreferable to either group. In general, Denmark and Germany referred a distinctly higher proportion of the otoliths to the "wide" category, a feature that is also brought out in the later compilation of the otolith data of the surveys (see section C IV).

For practical reasons it was not possible to circulate material for testing the comparability of the determination of K_2 and VS counts and maturity. At the meeting of the Herring Working Group in Hamburg, 1962, the participating scientists (including Belgium, Norway and Sweden) did maturities on a small sample of herring. The result is included in Appendix 1 but has little bearing on the material of the young herring surveys, because the determinations were not done by the people who did the routine analysis of the young herring samples. The small test did, however, again stress the necessity for a careful and unambiguous distinction between virgins and recovering spents.

(iv) Other meristic characters

Vertebral counts have been undertaken in all laboratories for many years, but K_2 countings were a new character to some of the laboratories involved in the surveys. A number of technical assistants from Germany and the Netherlands had an opportunity to count keeled scales on the same material in 1965. Only in one case out of a hundred was a different value obtained by the two teams for VS, and none in K_2 counts. On this evidence it would appear that the postulated difficulties of K_2-counting are overrated.

A VI Spring spawners

The main objective of these adolescent herring surveys was to identify the centres of abundance of pre-recruit herring in the North Sea and relate them to the major adult stocks. Spring-spawning herring are now of little commercial importance in the North Sea fisheries, and are poorly sampled for meristic characters. Accordingly the distribution and characters of spring-spawning adolescents were not analysed in the data collected. However, in preparing the data for the discriminant function analysis, it was essential that spring-spawning fish be identified so that they could be removed from the data to be analysed.

In the material collected on these surveys herring of spring-spawned origin were identified from their otolith characteristics. Einarsson (1951) described a distinction in the structure and appearance of the otolith nucleus between spring-spawning and summer-spawning herring in the Iceland area. Parrish and Sharman (1958) showed that for the most part spring- and summer/autumn-spawned fish from the North Sea could also be separated on the basis of the character of their otolith nuclei, but these authors extended Einarsson's analysis to otolith features other than those of the nucleus, to cope with the small proportion of otoliths having doubtful nucleus characteristics. For the material collected on these surveys the separation of the spring-spawned fish from those spawned in summer/autumn was carried out using the characters described by Parrish and Sharman. These are: (1) that the otoliths of fish of the spring group have opaque nuclei, whilst those of the summer/autumn group have hyaline nuclei; (2) otoliths of the winter/spring group have a finer texture and a smoother outline than those of the summer/autumn group; (3) the first growth zone is generally smaller amongst the spring group than amongst the summer/autumn group; (4) the winter zones are different in general appearance between members of the two groups; and (5) the antirostrum of the spring spawners is larger, relative to the rostrum, than in the summer/autumn spawners.

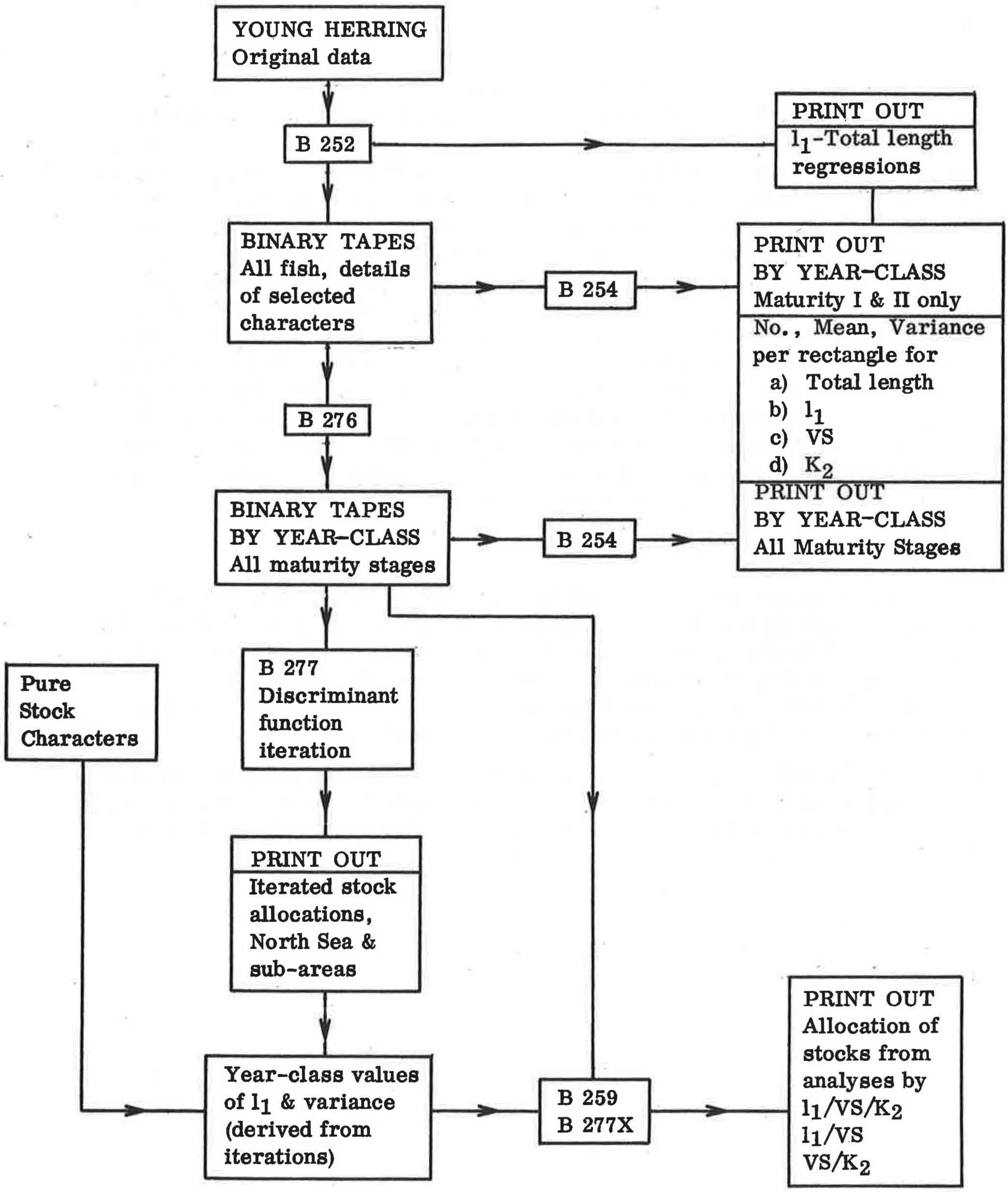
Wood and Foster (1966) have shown that fish from the Dunmore stock, which spawns from November to February off the south coast of Ireland, have a wide range of otolith types, some of which are consistent with summer/autumn and some with winter/spring types. Unfortunately a similar divergence in otolith types, although of a smaller magnitude, has been observed in some North Sea stocks. Einarsson (1951) found that although 85.6 per cent of Norwegian spring spawners had an opaque nucleus, the rest had hyaline centres of varying size, some of which closely resembled the nucleus of summer spawners. Conversely Postuma (1967), examining the 1964 and 1965 year-classes spawning at Whitby, Dogger and Sandettié, found numbers of herring spawning in autumn with an opaque type nucleus, even after grinding part of the otolith away. He gave the following percentages of opaque centres: Dogger 7.5, Sandettié 9.9, Channel 20.5.

Unfortunately, many of the criteria used in otolith typing demand a rather subjective judgement and, particularly where material is analysed by people working in different laboratories, it cannot be guaranteed that the criteria used are in all cases interpreted in the same way. If the classification into spring and autumn spawners has been based entirely on the otolith nucleus type (and this in effect is what has been done), it is not surprising that there have been a fair number of misclassifications, with some spring spawners being included in the material which has been subjected to the computer analysis. Some spring-spawned fish certainly appear to have been identified as autumn spawners from their otolith type. A considerable number of spring-spawned "long-shore" herring may have been included amongst the autumn spawners in the western area in the autumn of 1960. These have a fairly small hyaline nucleus, and although in fish of 2 years of age or more the growth pattern is quite distinctive in most cases, there are again some which are very similar to North Sea autumn spawners, and these often have 56 vertebrae, so their vertebral count does not help in their removal. These would have been misclassified as autumn spawners.

A VII The computer program

The computer used for the analyses was the Elliott 803 computer of the Ministry of Agriculture, Fisheries and Food, situated at Guildford, England. The preparation of the data proved to be a far more formidable task than had been foreseen in November 1963. In many cases the data sheets submitted were either incomplete or were not uniformly prepared. However, in March 1965 the data tapes were finally ready for commencement of the analysis.

The following scheme shows, in summary, the various programs and their main outputs, which were used to derive the final stock allocations. In Appendix 2 a more detailed description is given of the individual programs and their outputs.


YOUNG HERRING
Original data
B 252
PRINT OUT
l1-Total length regressions
BINARY TAPES
All fish, details of selected characters
B 254
PRINT OUT
BY YEAR-CLASS
Maturity I & II only
No., Mean, Variance per rectangle for
a) Total length
b) l1
c) VS
d) K2
B 276
BINARY TAPES
BY YEAR-CLASS
All maturity stages
B 254
PRINT OUT
BY YEAR-CLASS
All Maturity Stages
Pure Stock Characters
B 277
Discriminant function iteration
PRINT OUT
Iterated stock allocations, North Sea & sub-areas
Year-class values of l1 & variance (derived from iterations)
B 259
B 277X
PRINT OUT
Allocation of stocks from analyses by
l1/VS/K2
l1/VS
VS/K2

B CATCHES

B I Distribution of fish by age and length

The catches per hour fishing for each year-class in each statistical square are shown in Figures B I 1-12. Catches have been expressed in numbers of fish of autumn spawners only. The catch figures are sometimes based on only one haul, but more often on two or more hauls. In general the figure in a square represents the catch of one of the vessels, but in some cases the catches of different vessels have been combined (see section A III).

The charts are intended to illustrate the distribution and, by comparing the distribution of a year-class during the four surveys, the movements of the immature fish. It should be realized, however, that the information on the abundance of the immature herring yielded by the catches is limited and, perhaps, biased for the following reasons:

(i) The catch figures shown in the charts are in no way corrected for possible differences in power factors between ships. If differences in power factors existed, as is suggested in section B IV, the fact that the ships fished in separate areas will affect the comparability of the catch figures between areas.

(ii) The catchability of the herring may differ by area and by season, due for instance to differences in depth and temperature, which would again affect the comparability of the catch figures between areas and seasons. It is possible that the availability of the herring is less in the deeper water.

(iii) As is shown in section B IV, due to the high variance in the catch per haul the value of the catch of a single haul (or even the mean of two or three hauls) as an indication of abundance is very small. This objection, however, can be overcome by comparing the abundance in larger areas, thereby combining the information of several hauls and consequently reducing the variance.

In Figure B I 13 the relation between the two standard sub-divisions used in the analysis can be seen. The crudest sub-division is made by first dividing the area into two along the line of 2°E longitude, and then separating the eastern region into "north-east" and "south-east" along the line of 57°N latitude. The choice of the five sub-divisions is made in order to give regions in which the variance on catch per effort is low and of the same order.

Tables B I 1 a and b show estimates of abundance in these larger areas, obtained by adding up the average catches per square, raised to the total number of squares in the area. This procedure assumes that all mean catches per square are equally good estimates of abundance in an area.

From these considerations it follows that the data on catches by age-groups and seasons in Figures B I 1-12 can only be used as a rough guide for judging the abundance and thereby the distribution and movements of the immature herring. The following conclusions, which in general confirm those of other authors based on earlier material (Cushing 1962, Postuma *et al.* 1965), have been drawn from the figures and from Tables B I 1 a and b:

Table B I 1a Abundance estimates, by year-classes and seasons, in areas south-east, north-east and west (see Figure B I 13). The estimates were obtained by adding up the mean catches per hour fished per square in each larger area, multiplied by total number of squares in area/number of squares fished in area

Season	Year-class		Area West	South-east	North-east	Total
Spring 1960	1957	1	11 922	4 253	2 958	
		2	59/46	55/32	38/26	
		3	15 291	7 310	4 323	26 924
	1958	1	3 411	88 135	6 124	
		2	59/46	55/32	38/26	
		3	4 375	151 486	8 950	164 811
Autumn 1960	1957	1	4 079	1 979	21	
		2	59/48	55/40	38/15	
		3	5 014	2 721	53	7 788
	1958	1	1 426	39 829	10	
		2	59/49	55/40	38/15	
		3	1 717	54 765	25	56 507
	1959	1	1 080	2 632	-	
		2	59/48	55/40	38/15	
		3	1 328	3 619	-	4 947
Spring 1961	1957	1	1 508	490	174	
		2	59/44	55/41	38/29	
		3	2 022	657	228	2 907
	1958	1	4 170	7 020	3 091	
		2	59/43	55/41	38/30	
		3	5 722	9 417	3 915	19 054
	1959	1	3 747	21 133	3 627	
		2	59/43	55/41	38/30	
		3	5 141	28 350	4 594	38 085
Autumn 1961	1957	1	4 469	179	3	
		2	59/30	55/28	38/13	
		3	8 789	352	9	9 150
	1958	1	10 634	1 987	331	
		2	59/30	55/28	38/13	
		3	20 914	3 903	968	25 785
	1959	1	1 683	1 813	343	
		2	59/30	55/28	38/13	
		3	3 310	3 561	1 003	7 874
	1960	1	82	38 326	17 645	
		2	59/30	55/28	38/13	
		3	161	75 284	51 578	127 023

1 Catch per hour (numbers), in fished squares
2 Total number of squares/number of squares fished
3 2 x 1

Table B I 1b   Abundance estimates, by year-classes and seasons, in Areas 1, 2, 3, 4, 5 (see Figure B I 13). The estimates were obtained by adding up the mean catches per hour fished per square in each larger area, multiplied by total number of squares in area/number of squares fished in area

Season	Year-class		Area 1	Area 2	Area 3	Area 4	Area 5	Total
Spring 1960	1957	1	3 894	376	10 751	3 785	327	
		2	61/47	10/7	27/24	33/20	21/10	
		3	5 054	537	12 095	6 245	687	24 618
	1958	1	3 946	2 622	3 411	85 822	1 869	
		2	61/45	10/6	27/22	33/20	21/10	
		3	5 349	4 370	4 186	141 606	3 925	159 436
Autumn 1960	1957	1	353	8	4 254	1 464	-	
		2	61/34	10/6	27/25	33/26	21/11	
		3	633	13	4 594	1 858	-	7 098
	1958	1	406	2	1 203	39 654	-	
		2	61/36	10/6	27/25	33/26	21/11	
		3	688	3	1 299	50 329	-	52 319
	1959	1	-	-	1 080	2 595	37	
		2	61/35	10/6	27/25	33/26	21/11	
		3	-	-	1 166	3 294	71	4 531
Spring 1961	1957	1	369	45	1 212	200	346	
		2	61/50	10/7	27/19	33/25	21/13	
		3	450	64	1 722	264	559	3 059
	1958	1	734	3 048	2 913	4 072	3 514	
		2	61/50	10/7	27/19	33/25	21/13	
		3	895	4 354	4 140	5 375	5 677	20 441
	1959	1	1 103	3 625	2 255	19 509	2 015	
		2	61/50	10/7	27/19	33/25	21/13	
		3	1 346	5 179	3 205	25 752	3 255	38 737
Autumn 1961	1957	1	114	-	4 472	65	-	
		2	61/21	10/5	27/19	33/16	21/10	
		3	331	-	6 355	134	-	6 820
	1958	1	26	330	11 036	1 556	4	
		2	61/21	10/5	27/19	33/16	21/10	
		3	76	660	15 683	3 209	8	19 636
	1959	1	149	341	1 537	1 806	7	
		2	61/21	10/5	27/19	33/16	21/10	
		3	433	682	2 184	3 725	15	7 039
	1960	1	348	17 298	89	37 880	438	
		2	61/21	10/5	27/19	33/16	21/10	
		3	1 011	34 596	126	78 128	920	114 781

1 Catch per hour (numbers), in fished squares
2 Total number of squares/number of squares fished
3 2 x 1

(i) The youngest age at which a year-class of herring was encountered during the surveys was approximately 1 year, i.e. as 0-group herring in autumn (Figures B I 1-2). During the preceding spring cruise, when these fish were about $\frac{1}{2}$ year old, they were still predominantly in the larval stage and were therefore not retained by the nets used.

(ii) The area of distribution of the young herring increased after the 0-group stage, as is indicated in Table B I 2, which shows the rectangles in which herring were present (excluding squares with less than 10 fish) as a percentage of the total squares fished. The area in which a year-class occurred increased markedly between 0-group, autumn and I-group, spring. Thereafter the evidence on the individual year-classes suggests that the area of distribution might become smaller again.

Table B I 2 Area covered by each year-class per season, expressed as percentage of the rectangles fished in which herring were present. Rectangles with less than 10 fish were considered to contain no herring

	Year-class				
	1957	1958	1959	1960	Mean
0-group autumn			7	28	17
I-group spring		51	30		40
I-group autumn		24	23		23
II-group spring	55	35			45
II-group autumn	30	35			32

(iii) In the distribution of the immature herring in the North Sea a number of loosely defined centres can be distinguished, which are most probably determined by the geographical and hydrographical situation. The main and most constantly occupied centres were found in the central North Sea, to the east and west of the Dogger Bank. Other centres occurred in the Skagerak, the Moray Firth, and the eastern part of the Southern Bight. II-group herring, both in spring and autumn, were also found in the Egersund area, in the eastern part of the northern North Sea. Only in one case were I-group fish found in numbers in the Egersund area (1958 year-class, spring).

(iv) A consistent difference existed between the spring and autumn distribution of the immature herring, the area occupied in spring being always larger than that in autumn. The spring distribution included almost all the areas mentioned under (iii), but in autumn the area in which the herring were found was mostly restricted to the regions around the Dogger Bank, with an extension to the northern North Sea.

The sole exception to this rule was in the distribution of the 0-group herring in autumn, which occurred in the more coastal centres, e.g. in the Skagerak and the Southern Bight (Figures B I 1-2).

It is likely that the difference between the autumn and spring distributions, which can also be observed as a lower percentage of occupied rectangles in autumn in Table B I 2, is caused by an avoidance of areas with higher temperatures by herring older than 1 year.

(v) With increasing age the immature herring moved in a generally north-westerly direction, which is demonstrated by the following descriptio^r^ based on Figures B I 1-10:

(a) 0-group herring in autumn were mainly found east of the Dogger.

(b) I-group herring in spring still seemed to be most abundant in the area east of the Dogger, but also occurred in large numbers to the west of the Dogger and, in the case of the 1958 year-class, in the Egersund area.

(c) I-group herring in autumn were in about the same area as in spring, although the area of distribution was somewhat contracted. The 1958 year-class was absent in the Egersund area.

(d) II-group herring in spring, of which a large proportion had become adult (see section C V), had again approximately the same pattern of distribution as in the preceding spring, in that most of the fish were found west and north-west of the Dogger Bank and a small part was still situated east of that bank.

(vi) The main migration of the immatures took place between autumn and spring, as follows from the description under (v).

Between September and March the young herring seemed to migrate mainly to the west and north-west, passing the Dogger probably both to the north and to the south. In the eastern North Sea, however, the herring also moved to the east and south, as follows from the larger extension of the spring distribution.

Between March and September movements were restricted to those away from the coastal and shallower areas. In addition, displacements on a small scale could be observed in the areas east and west of the Dogger, again in a general north-westerly direction.

(vii) Differences occurred in the distribution and migration pattern between year-classes, as has been already indicated. The 1957 year-class, for instance, tended to move at an earlier age to the deeper waters north-west of the Dogger than did the 1958 year-class, whereas the 1959 year-class took an intermediate position in this respect. The later migration of the 1958 year-class could be connected with the fact that this year-class proved to be a predominantly southern one after recruitment, spawning late in the season, mainly on the Dogger and in the Southern Bight (see section D III).

Differences between the four year-classes which were encountered during the surveys as immature fish were not confined to their distribution and migration, but were also apparent in their relative abundances. This is demonstrated, for instance, in Table B I 1 b, where an estimate of abundance in the whole North Sea has been computed by adding up the estimates of the different areas. Comparing these abundance indices for each age-group in the same season, it is clear that the 1960 year-class as 0-group in autumn was far more abundant than the 1959 year-class

(25.3 x) and had also a far larger area of distribution (see Table B I 2). Comparing the 1958 and 1959 year-classes as I-group fish in both spring and autumn, the 1958 year-class appears as the stronger one, again both in abundance (autumn 7.4 x, spring 4.1 x) and in the size of the area of distribution. The relative strength of the 1957 and 1958 year-classes is less clear: in spring as II-group fish the 1957 year-class appears to be the stronger one (1.2 x) on both criteria, whereas in autumn the 1958 year-class seems to be more abundant (2.8 x) and to occupy a larger area.

It should be noted here that on all occasions a relatively higher abundance coincided with a larger area of distribution.

From the relative abundances at comparable seasons, as described above, the relative strength of the four year-classes encountered during the young herring surveys could be estimated as follows:

1957	4.5
1958	5.8
1959	1.0
1960	25.3

These figures should be compared with the estimates of the year-classes as adult fish, as is done in section D III.

The changes of the mean lengths of the autumn-spawning herring of all maturity stages during the four surveys have been shown schematically by age-groups in Figures B I 14-24 for each 30 x 30 nautical mile statistical square. With the exception of the 0-group herring the data indicate that:

(i) the mean length of fish of the same age-group increased to the north and west, so that the smallest fish were found in the south-east corner of the area of distribution, near the German and Danish coast, whereas the larger fish of a year-class occurred west and north of the Dogger Bank;

(ii) superimposed on the trend in the length described under (i) a tendency existed for the fish to be smaller in coastal areas.

The 0-group herring caught in autumn (Figures B I 14-15) do not show, in their rather limited area of distribution, any clear trend in length. Table B I 3 gives a summary of the numbers of observations, mean length and variance for the western, north-eastern and south-eastern North Sea.

Table B I 3 Number of observations, mean length (cm) and variances for North Sea areas

Age	Year-class	Survey		West	North-east	South-east
0	1959	Autumn 1960	N	152	0	299
			$\bar{L}$	14.6	-	14.3
			s^2	1.84	-	1.37
	1960	Autumn 1961	N	64	254	1 918
			$\bar{L}$	13.5	13.1	14.6
			s^2	0.82	2.53	1.45
I	1958	Spring 1960	N	496	1 091	3 035
			$\bar{L}$	16.8	16.4	14.4
			s^2	3.30	4.07	4.40
	1959	Spring 1961	N	489	477	701
			$\bar{L}$	16.4	16.4	16.2
			s^2	2.47	2.90	2.12
	1958	Autumn 1960	N	420	8	1 839
			$\bar{L}$	22.2	22.4	18.7
			s^2	1.73	6.93	2.72
	1959	Autumn 1961	N	208	16	192
			$\bar{L}$	21.0	20.9	21.1
			s^2	2.91	1.67	2.65
II	1957	Spring 1960	N	1 254	618	725
			$\bar{L}$	21.1	21.1	20.0
			s^2	2.86	3.23	1.99
	1958	Spring 1961	N	571	354	1 034
			$\bar{L}$	21.9	20.6	20.5
			s^2	3.18	4.73	3.04
	1957	Autumn 1960	N	1 226	21	308
			$\bar{L}$	24.3	26.3	24.2
			s^2	1.84	1.29	2.13
	1958	Autumn 1961	N	1 306	5	990
			$\bar{L}$	24.9	23.6	24.2
			s^2	1.61	4.55	1.81
III	1957	Spring 1961	N	234	112	107
			$\bar{L}$	25.0	25.3	23.6
			s^2	1.48	1.47	1.44

Table B II 1 Numbers of herring caught per 100 hours trawling in the Bløden fishery

Area (see Fig. B II 2)	Numbers caught (millions)				Total all year-classes	Hours trawling	Number of statistical squares
	March/April 1960						
	Year-class						
	1958	1957	1956	older			
I	5.02	1.24	0.11	-	6.37	596	8
II	2.37	1.41	0.30	+	4.08	5 913	10
IV	7.18	-	-	-	7.18	477	3
VI	2.37	0.03	0.07	0.01	2.49	4 108	6
VII	0.40	0.21	+	-	0.62	160	6
VIII	0.50	0.24	0.01	-	0.75	2 754	9
XI	-	0.04	0.30	+	0.34	2 649	-
	September 1960						
	Year-class						
	1959	1958	1957	older			
I	1.03	4.22	-	-	5.25	36	8
III	-	1.81	0.09	-	1.90	2 885	4.5
IV	-	1.34	0.22	-	1.56	1 388	3
V	-	2.22	0.11	-	2.33	196	4
VI	-	1.88	0.30	-	2.18	380	6
	March/April 1961						
	Year-class						
	1959	1958	1957	older			
I	1.59	0.47	-	-	2.06	432	8
II	0.15	3.01	0.14	-	3.29	4 900	10
IV	0.16	0.59	0.04	-	0.78	174	3
VI	0.43	1.62	0.10	-	2.15	1 183	6
VIII	0.09	0.36	0.02	-	0.48	499	9
	September 1961						
	Year-class						
	1960	1959	1958	older			
I	13.51	-	-	-	13.51	2 100	8
III	2.21	0.10	0.47	0.02	2.80	9 108	4.5
IV	2.02	0.21	0.36	-	2.60	126	3
V	1.90	0.09	0.40	0.01	2.41	753	4
VI	2.70	-	-	-	2.70	2 266	6
VII	3.75	0.18	0.80	0.03	4.75	182	6

B II Seasonal abundance estimates and distribution of the Bløden fishery

Catch and effort data of the Danish fishery on young immature herring in the eastern North Sea have been collected since the first ICES tagging experiment in 1957. The treatment of the data is described in Aasen et al. (1960).

This industrial fishery started on a very small scale in 1948 and reached a peak in 1958. In 1960 the effort was reduced by about 20 per cent due to a fishermen's strike in August, which is usually the month of highest catches. In the same year the rapid increase in Peruvian production led to a drop in fishmeal prices on the international market and to a reduction in the Danish industrial fishing effort for some years afterwards.

The distribution of the fishery and estimates of the abundance of the year-classes involved are shown in Figures B II 1-4 and Table B II 1. For abundance estimation the North Sea is divided into the areas shown in Figure B II 2. For each area the catch in weight is converted to total number of herring caught. This total is then split by year-classes according to market measurements and age-length keys for that particular area. In Table B II 1 the abundance of each year-class is given by areas as numbers caught per 100 hours pair trawling.

The Bløden fishery is mainly confined to areas I-VI. In years of great abundance the fishery may extend to areas VII and VIII, but usually these areas only occur in the herring statistics when herring are landed as a by-catch from fisheries on other species, such as sandeels.

The mean abundance figures for each year-class for each season in the fishery have been calculated from Table B II 1 by weighting the abundance figure in each area by the appropriate number of 30 x 30 nautical mile statistical squares; these mean figures are shown in Table B II 2.

Table B II 2 Year-class abundances for each survey

	Year-class						
	1960	1959	1958	1957		1956	Total
Spring 1960	-	-	2.54	0.66		0.10	3.30
Autumn 1960	-	0.32	2.59	0.13		-	3.04
Spring 1961	-	0.50	1.35	0.06		-	1.92
Autumn 1961	5.41	0.08	0.30		0.01		5.80

The year-classes are difficult to compare because they are of different ages and so at different stages of immigration to and from the Bløden area. It is evident, however, that the 1960 year-class was already very abundant as 0-group in the autumn of 1961, whereas the 1959 year-class was very poor in all seasons.

A rough estimate of the relative strengths of year-classes can be obtained by comparing the abundance estimates of these year-classes at the same age and in corresponding seasons; the indices thus compared are shown by connecting lines in the table above. The comparison between the 1958 year-class in the autumn of 1960 and the 1959 year-class in the autumn of 1961 has been omitted because of doubts about the latter abundance index. Taking the 1958 year-class as 100 the relative strengths of the year-classes are then:

1957	46
1958	100
1959	20
1960	315

These estimates should be compared with those obtained from the catches of the research vessels described in the preceding section (page 16); they show a fairly close agreement. In section D III the two sets of year-class estimates will be compared with those obtained in the adult North Sea stocks.

B III Distribution of the herring in relation to the hydrographical situation

During the surveys temperature and salinity observations were made at each trawl station and sometimes at other locations as well. The results of a study on these observations were published separately some years ago (ICES 1966).

Although a thorough study of the connections between these hydrographical factors and the distribution of the immature herring was considered to be outside the scope of the present study, an attempt has been made to detect possible relationships.

The pattern of the juvenile herring distribution did not show any clear relationship with factors such as surface temperature, salinity, density or thermal layering. However, a comparison of the distribution of the juvenile herring with bottom temperatures, as shown in Figures B III 1-4, suggested some connections. It should be noted that the density indices in the figures are the sums of the catches of two year-classes, which have been corrected to the same average level.

Both of the autumn surveys indicated that the immature herring were distributed mainly in or near areas where the temperature changed rapidly over short distances. This distribution in transition areas between relatively warm and cold bottom water was most clear in the autumn of 1961, but could also be observed in the autumn of 1960. There were no indications that in this season the herring tended to stay in water of a certain temperature. On the contrary, in both years they were found in water ranging in temperature between 8 and 16^{o}C, which is almost the whole range of temperature to be found in the North Sea in September.

In spring no such pronounced transition areas were present, but in some cases the herring did tend to concentrate again in weaker transition zones, as for instance near Helgoland, in the south-eastern North Sea, in the spring of 1960. But the two spring distributions show a remarkable difference, possibly related to differences in the temperature and/or salinity pattern. In the spring of 1960 the juvenile herring were spread all along the deep-water channel in the north-eastern North Sea in the Egersund-Skagerak area, in a general north-westerly direction. This concentration was lacking in the spring of 1961, when juvenile herring were only found in quantity in the Skagerak area proper. A comparison of the hydrographical situation in the two years shows that in 1960 a tongue of colder, low-salinity North Sea water protruded in that area in a north-westerly direction, whereas in the spring of 1961 a tongue of warmer, high-salinity Atlantic water pointed to the south-east.

Figure B III 1 strongly suggests that the distribution of the juvenile herring in the Egersund area in the spring of 1960 was closely associated with the tongue of cold North Sea water in that area. There were no indications that the herring tended to stay in water of a given temperature and/or salinity.

Although no clear relationships between the hydrographic situation and the distribution of the juvenile herring were available, considering the whole of the North Sea it seems as if in more restricted areas the herring were associated locally with certain hydrographic features.

B IV Fishing power and abundance estimates

One of the objectives in carrying out these surveys was to locate herring nursery areas throughout the North Sea and to evaluate their relative importance. In addition, it was hoped that it would be possible to identify recruits to the three major North Sea stocks within these nursery areas, and to follow their movements with age up to recruitment to the adult stocks. Both these objectives required that abundance estimates were obtained.

A first attempt at obtaining an overall picture of herring abundance throughout the area surveyed can be made by plotting the catch per unit time of a year-class, as has been done in section B I both for statistical squares and for larger areas. This approach assumes (a) that all the ships had equal fishing power, (b) that the fishing power did not change between seasons, and (c) that there were no differences in fishing power between areas. In this context fishing power is defined as:

$$p = \frac{c/f}{D},$$

where p is the fishing power, c is the total catch, f is fishing time, and D is the density of fish within the area.

In Appendix 3 a model has been constructed which attempts to take account of differences in fishing power between ships and cruises. For this purpose the total area surveyed has been divided into five sub-areas (see Figure B I 13) which show a reasonable degree of uniformity of herring density and contain a large number of hauls. Assuming a constant fishing power for the Bløden fishery in the four seasons, the catch per effort of the Bløden fishery can be taken as a measure of the true relative stock density, and seasonal changes in research vessel fishing power can be evaluated. This analysis showed differences in fishing power both between ships and between seasons, as may be seen in Table 4 (Appendix 3).

There was a very wide range of fishing power between ships in the spring of 1960, which might be due to the fact that for this first cruise no ruling was given as to whether hauls should be made randomly or guided by echo-sounder. On the three subsequent cruises differences between ships within a cruise and between cruises were in general of a much smaller order.

In Appendix 3 a second model has been used in which the fishing powers of all ships were taken to be equal, but fishing power was assumed to vary between seasons, referring again to the Bløden catch per unit effort. The results of this analysis showed the fishing power of the research fleet to be as follows during the four cruises:

Spring 1960	Autumn 1960	Spring 1961	Autumn 1961
6.447	3.615	3.450	2.802

Again, the fishing power in spring 1960 was found to be considerably higher than during the other three cruises, which showed negligible differences in fishing power

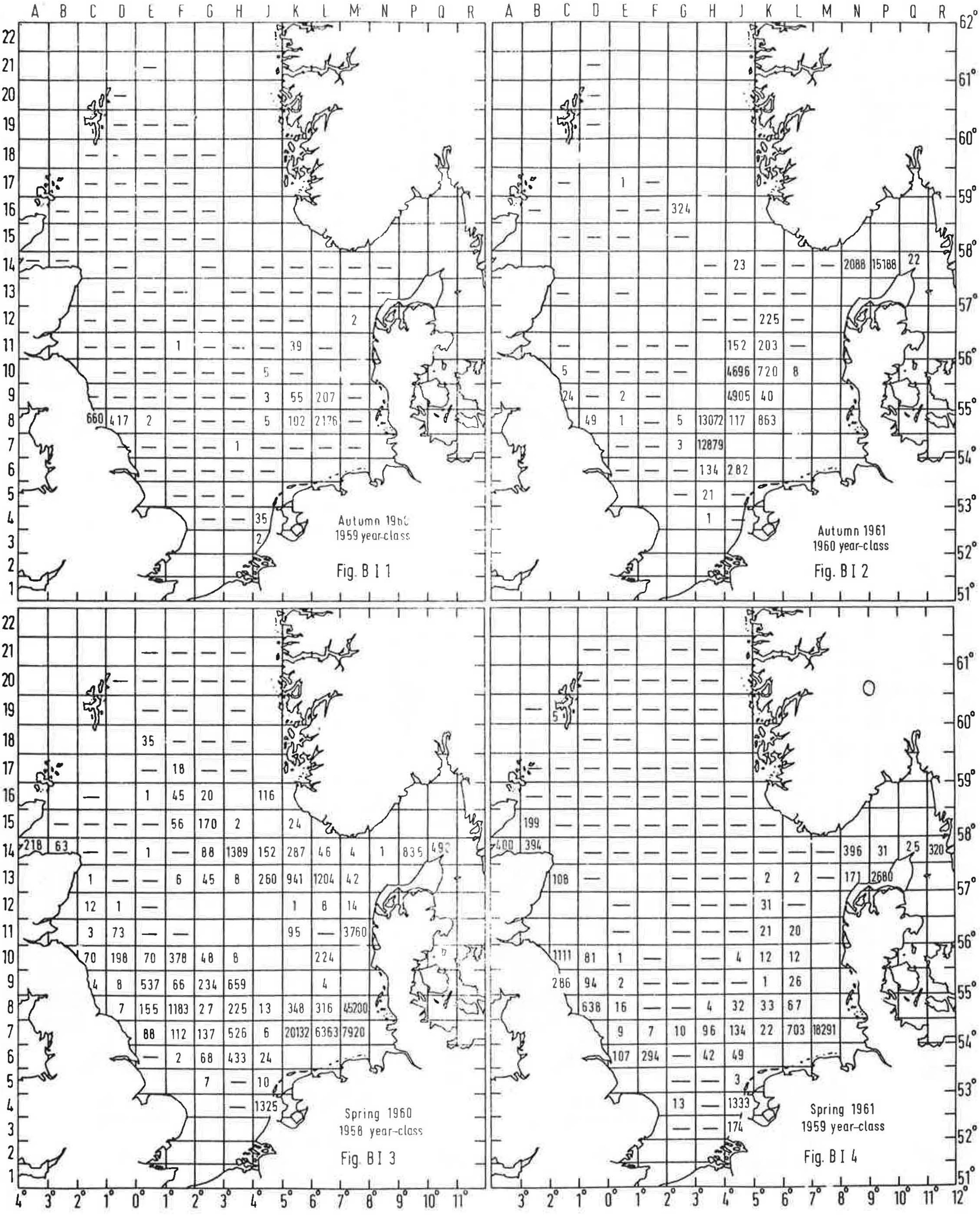
From the two models estimates of abundance have been obtained, which are shown in detail in Tables 6 and 9 of Appendix 3. A summary of these tables, together with the abundance estimates based on the assumption that fishing power does not vary between ships, seasons or areas (Table B I 1 b), is given below:

Survey	Estimate*	Year-class			
		1960	1959	1958	1957
Spring 1960	a	-	9	44 664	36 363
	b	-	1	16 355	3 253
	c	-	-	159 436	24 618
Autumn 1960	a	-	807	10 600	2 501
	b	-	805	10 491	2 261
	c	-	4 531	52 319	7 098
Spring 1961	a	-	6 918	4 953	931
	b	-	6 602	4 468	832
	c	-	38 737	20 441	3 059
Autumn 1961	a	90 610	2 263	5 866	2 281
	b	35 355	1 446	6 539	2 685
	c	114 781	7 039	19 636	6 820

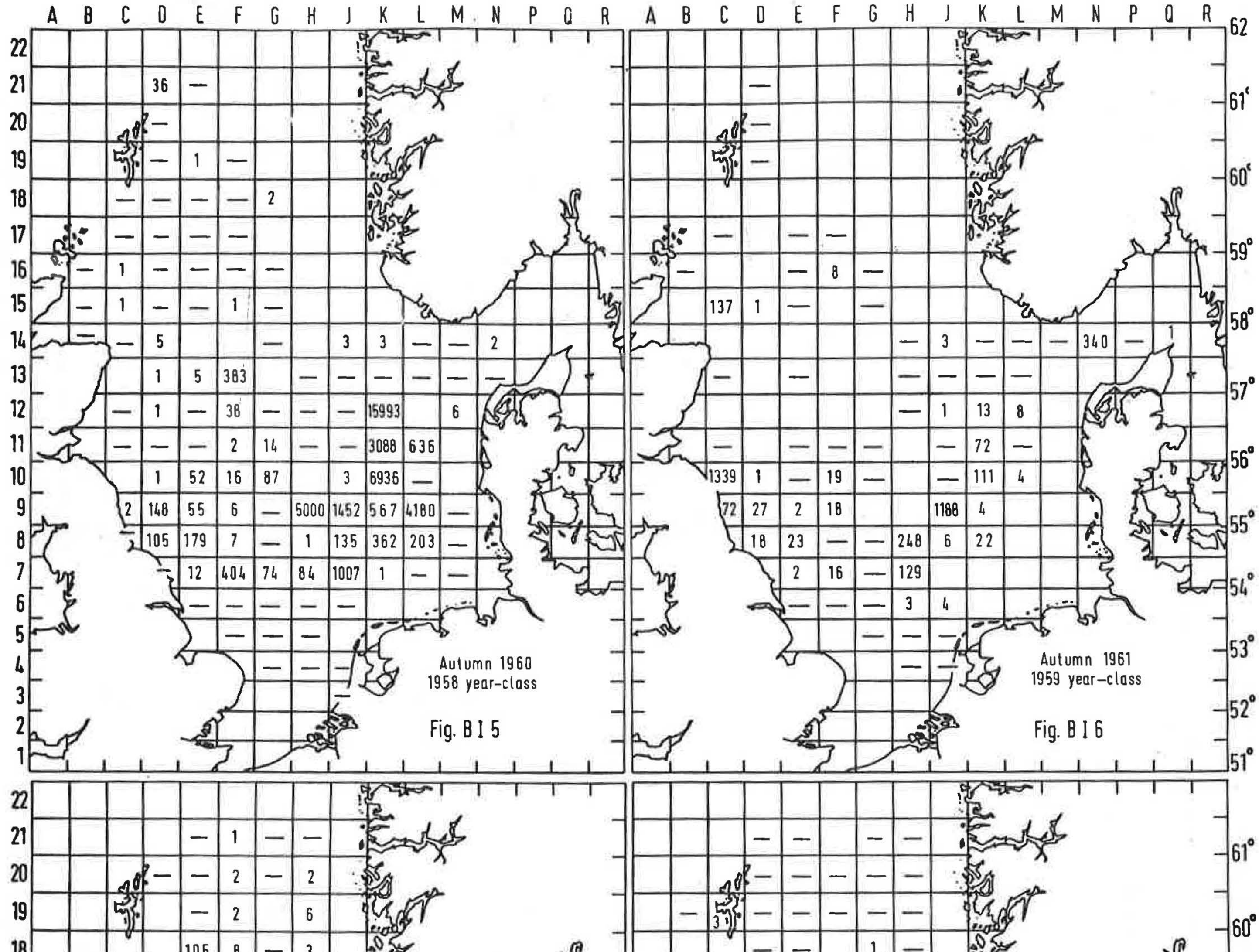
*estimate a = using ship power and season power factors
b = using season power factors only
c = no power factors (Table B I 1)

An additional difference between the three estimates is that for estimates a and b the assumption was that all catches within a sub-area (see Figure B I 13) are equally good estimates of stock density, whereas method c assumes catches to be valid only within a statistical square.

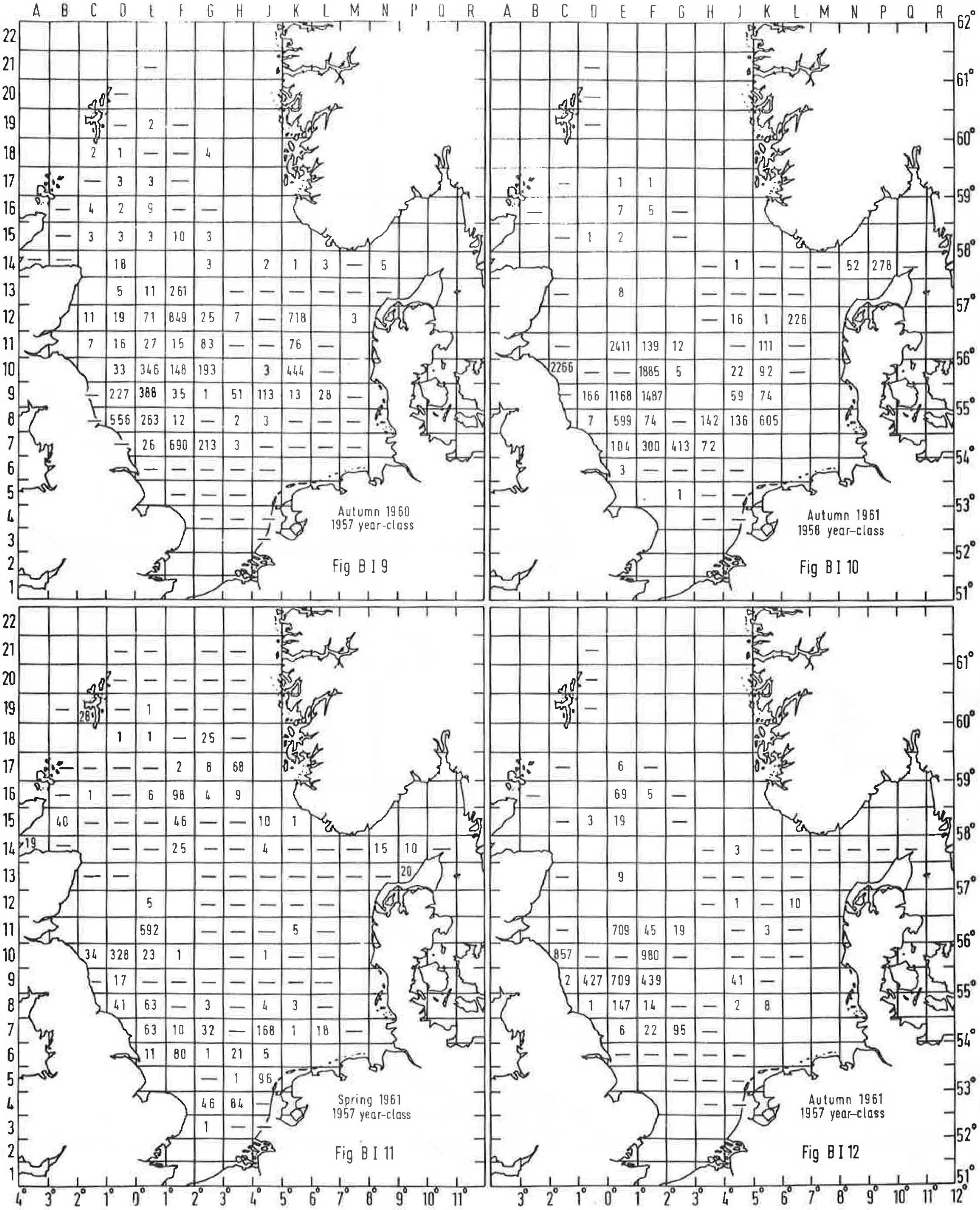
The three models give numerically very different answers, but the significance of the differences cannot be estimated. However, for the first two estimates it is possible to calculate their variances in contrast to the latter one where no estimate of variance can be made.



Figures B I 1-4 Catches per hour fishing (numbers of fish).



Figures B I 5-8 Catches per hour fishing (numbers of fish).



Figures B I 9-12 Catches per hour fishing (numbers of fish).

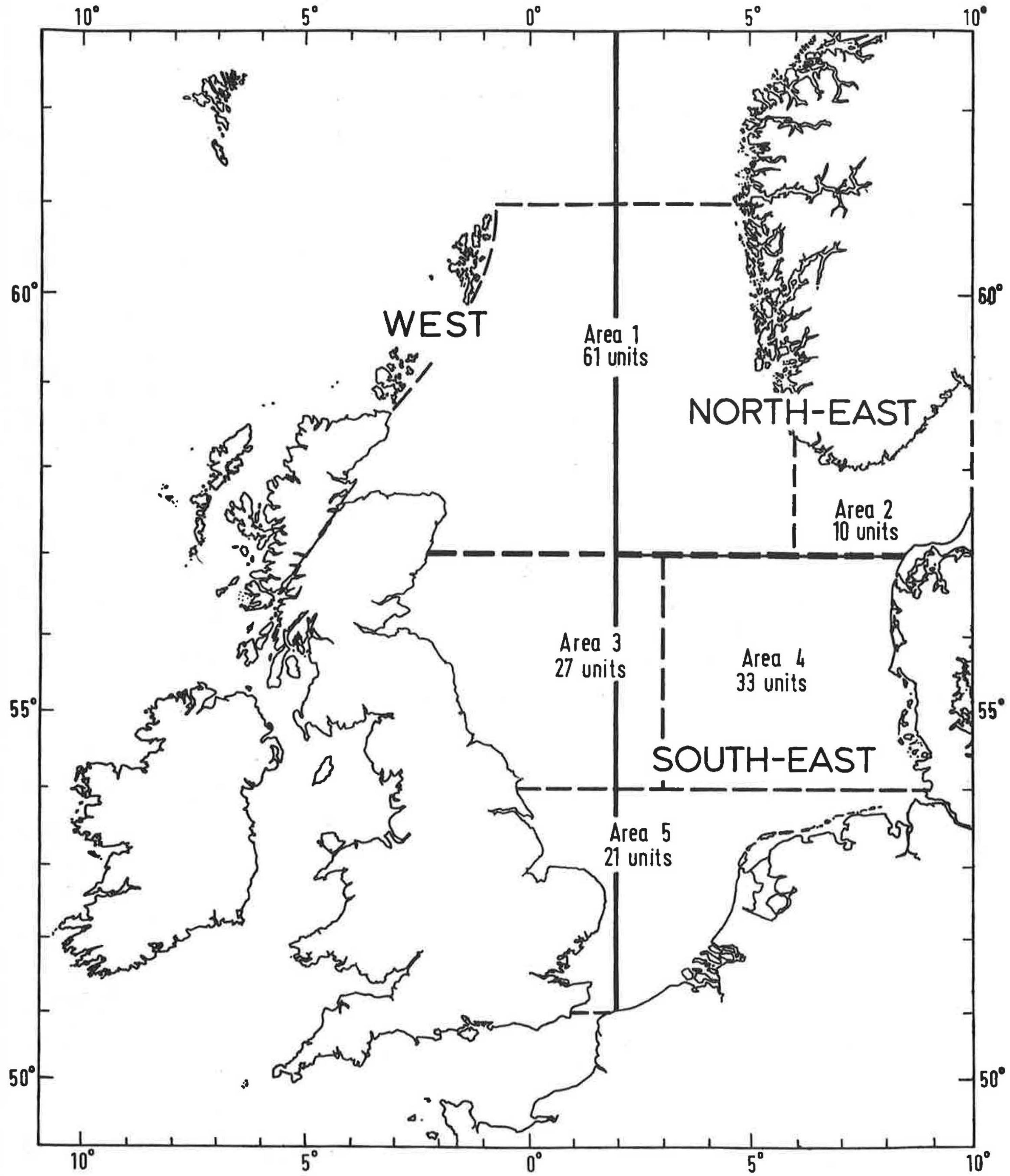
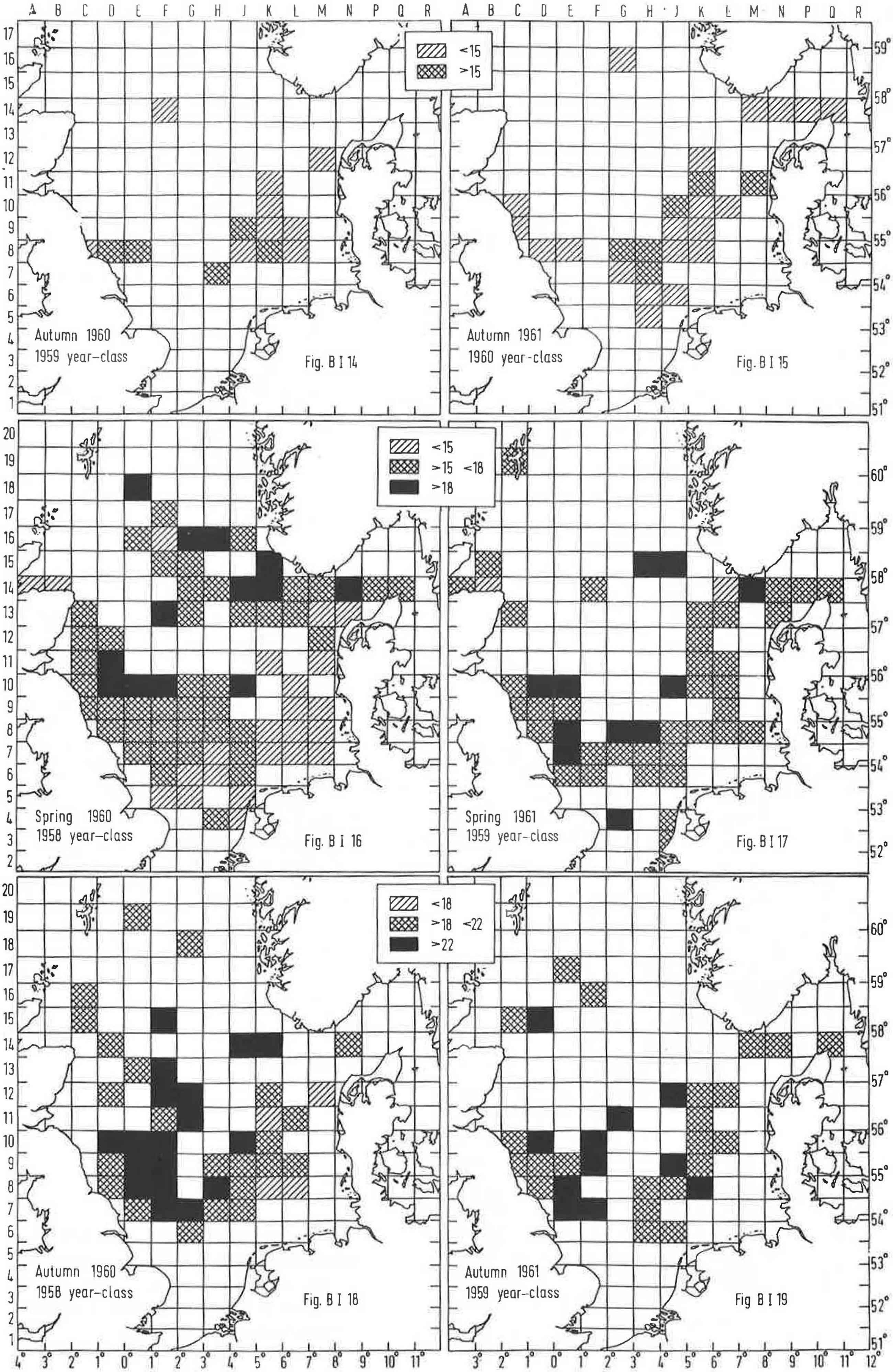
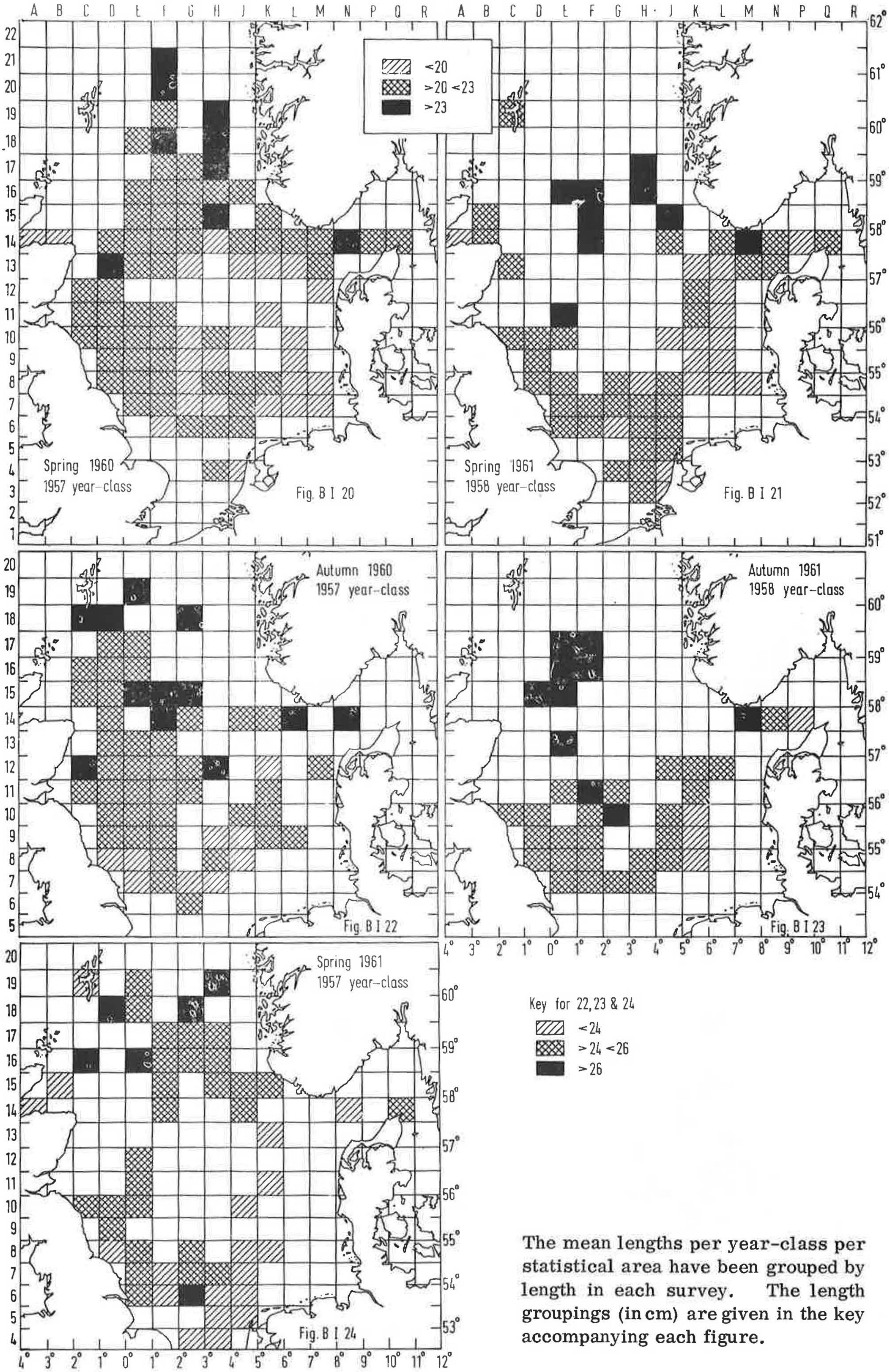


Figure B I 13 **Subdivisions of the North Sea used in the analyses.**

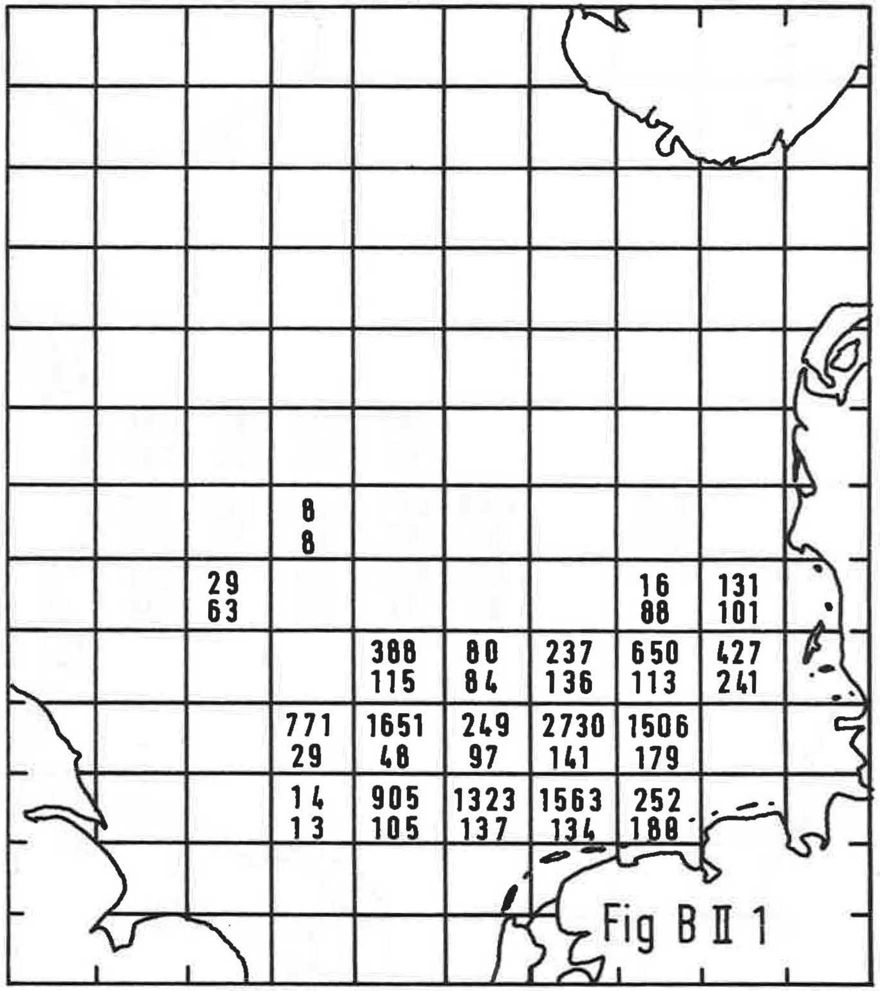


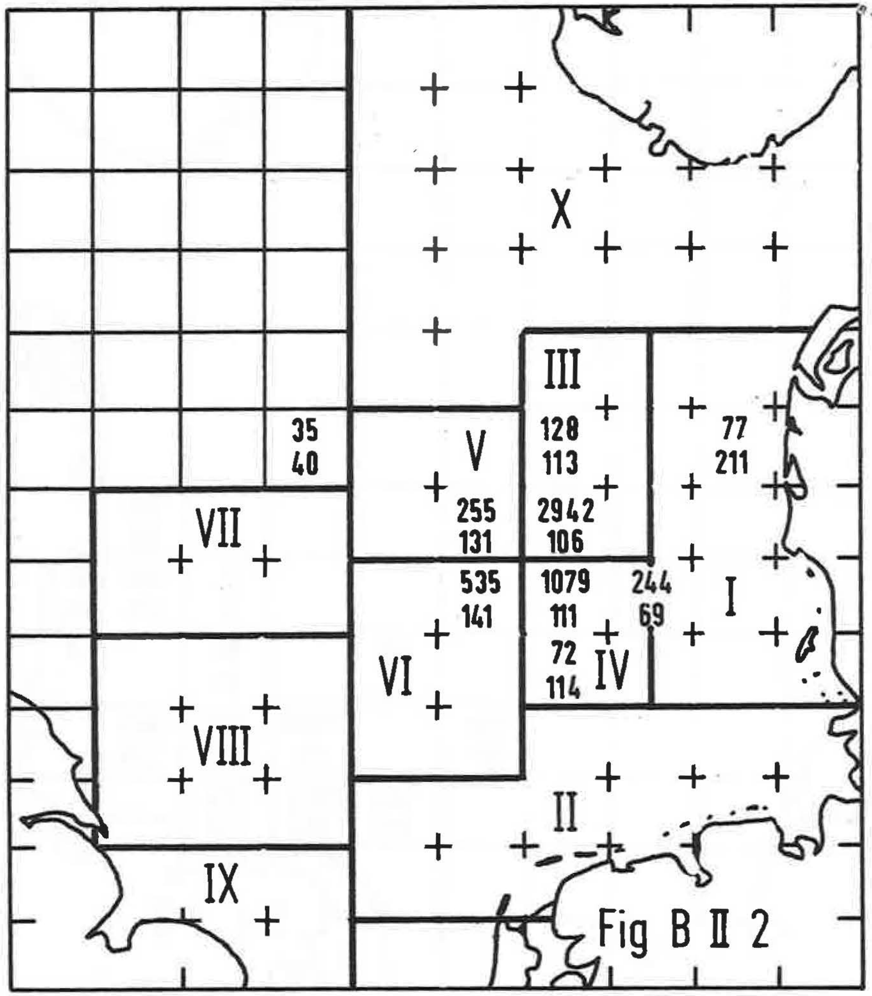
Figures B I 14-19 **Relative changes in length (cm) by areas (see note on Figures B I 20-24).**



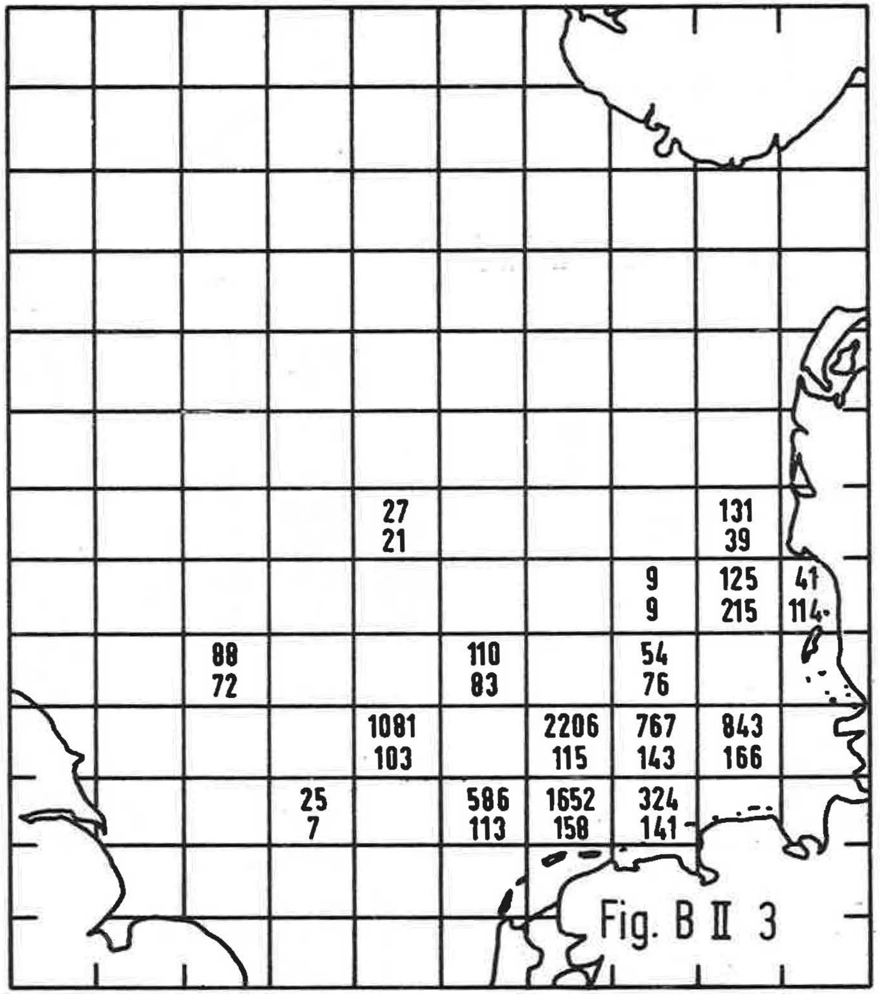
The mean lengths per year-class per statistical area have been grouped by length in each survey. The length groupings (in cm) are given in the key accompanying each figure.

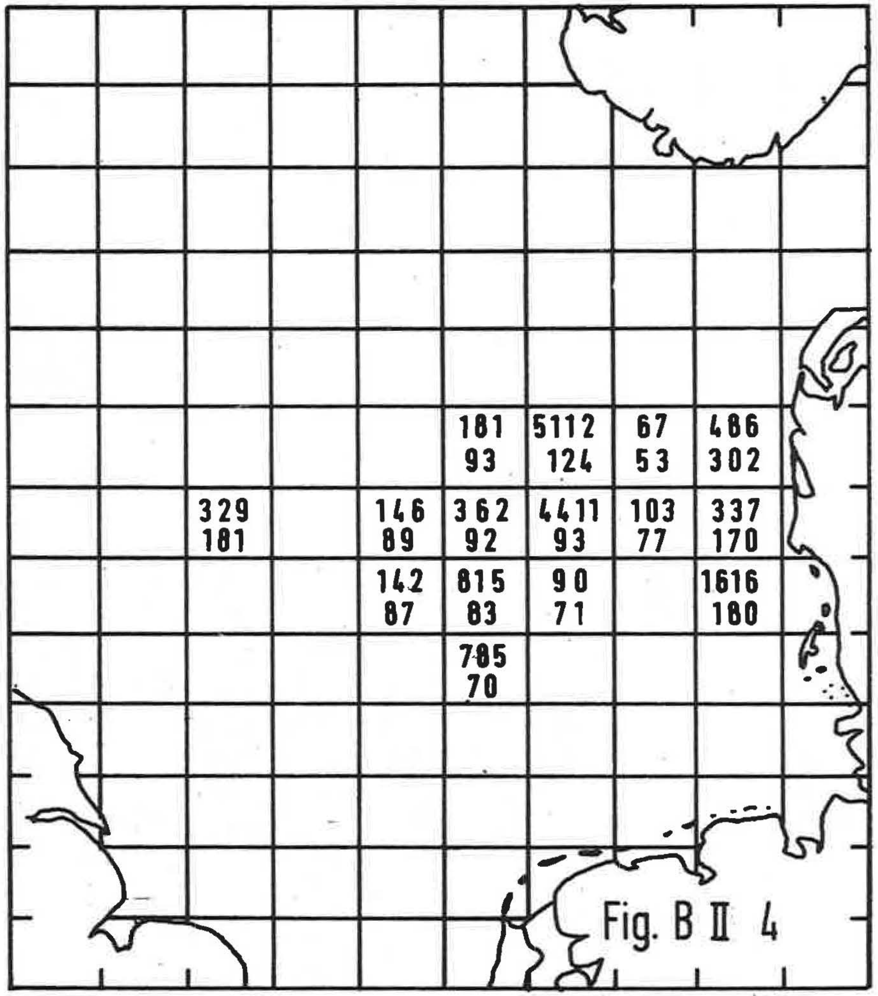
Figures B I 20-24 Relative changes in length (cm) by areas.



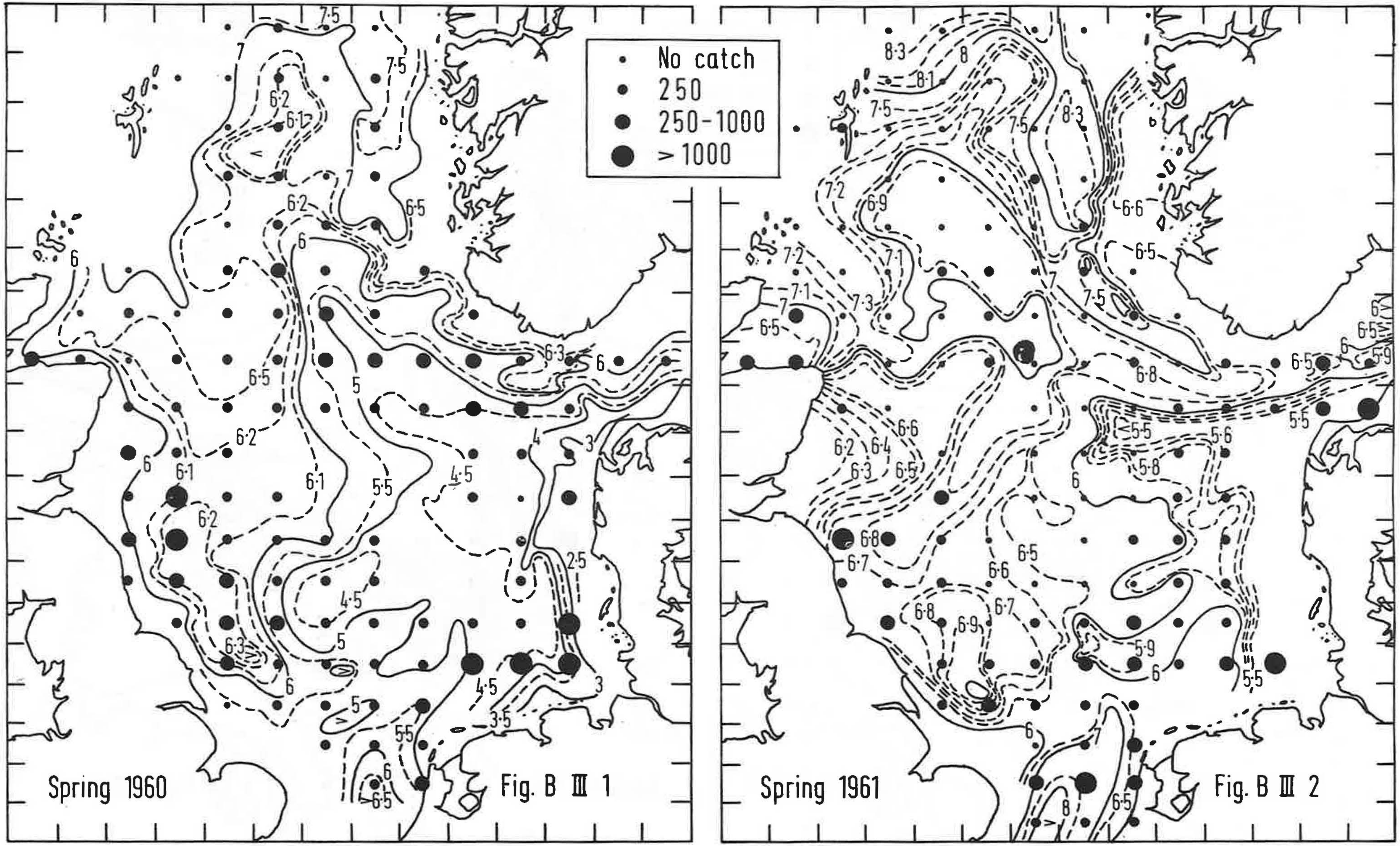


Figures B II 1 and 2 Distribution of the Danish young herring fishery (landings at Esbjerg), in 1, spring 1960; 2, autumn 1960. Top figures are total catch in tons, bottom figures are catch in tons per 100 hours pair trawling. In 2, the Danish Bløden statistical areas are shown by Roman numerals.

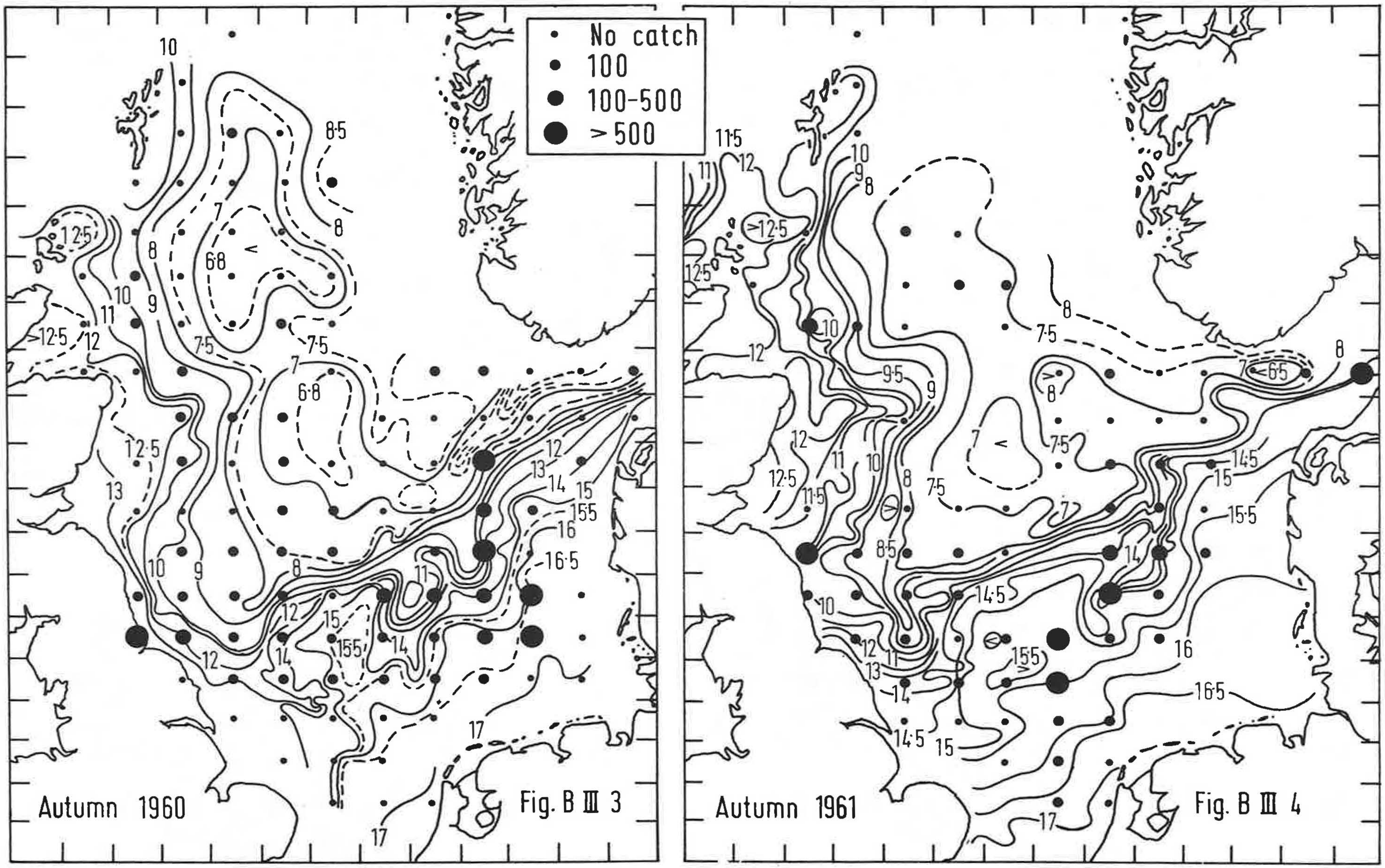




Figures B II 3 and 4 Distribution of the Danish young herring fishery (landings at Esbjerg), in 3, spring 1961; 4, autumn 1961. Top figures are total catch in tons, bottom figures are catch in tons per 100 hours pair trawling.



Figures B III 1 and 2 **Distribution of the catches of young herring (numbers of fish), in relation to bottom temperatures.**



Figures B III 3 and 4 **Distribution of the catches of young herring (numbers of fish), in relation to bottom temperatures.**

C CHARACTERS

C I Definition of Pure Stocks

For the purpose of defining the Pure Stocks in the adult, autumn-spawning North Sea herring, the concept of three stocks (Buchan, Bank and Downs), described in the report of the North Sea Herring Working Group (ICES 1965), has been adopted.

The Buchan herring have been defined as the herring group spawning off the Scottish north-east coast in August-September, whereas the herring spawning in August-October in the central North Sea, along the English north-east coast and on the slopes of the Dogger, have been defined as the Bank stock. Finally, to the Downs group belong the herring which spawn in the eastern English Channel and Southern Bight.

Information on the characters used to discriminate between the Pure Stocks in the immature herring (l_1, VS and K_2) has been obtained mainly from routine sampling on the spawning herring of the three stocks, carried out in England, Germany, Holland and Scotland.

In addition to l_1, VS and K_2, data on one other character of the Pure Stocks (otolith types) have been collected systematically, and these were used to test the results of the discriminant function analysis. This character will be treated in sections C IV and E II.

The characters of the three stocks have been derived mainly from 3-year-old spawning fish (maturity stage VI) of the year-classes 1957, 1958, 1959 and 1960. In the cases of VS and K_2, where the amount of data on the 3-year-old spawners was considered to be too small, older fish and fish of maturity stages V and VII have been included. In particular the K_2 values for the 1957 year-class in the Bank and Buchan stocks include older fish. The means and variances of l_1, VS and K_2, obtained by combining all information on these characters of the Pure Stocks, are shown in Table C I 1. The values have been obtained by taking the grand mean of all the available information for each of the Pure Stocks. The table demonstrates that the l_1s of the Downs and Bank herring are based on a fairly large number of fish. The Buchan data are mainly derived from drift-net catches, in which ripe herring were scarce; the unusually rough bottom on the Buchan grounds does not permit bottom trawling for herring.

The l_1 values observed in the Pure Stocks and shown in Table C I 1 have not been used in the discriminant function analysis, because there were indications that these values were biased as compared with the standard l_1 values used in the immature stocks. This problem has been fully discussed under section C II. Instead, standard l_1s have been computed, based on the l_1/total length relationship per year-class and length data on 3-year-old spawners.

Figure C I 1 shows the l_1/total length regression for the Pure Stock of the 1958 year-class of Downs herring. It is considered that the total length distribution represents the true distribution of the population, with a random error added. The variance of Y about the regression line is considered to include the variance due to the biases in l_1 determination.

Table C I 1 Means, variances and number of observations of total length, l_1, VS and K_2 in the Buchan, Bank and Downs stocks for the year-classes 1957, 1958, 1959 and 1960. The values have been obtained by averaging data from English, German, Scottish and Dutch sources (all lengths in cm)

	Bank			Downs			Buchan		
	$\overline{X}$	s^2	No.	$\overline{X}$	s^2	No.	$\overline{X}$	s^2	No.
	1957 year-class								
Total length	24.98			24.05			25.35		
l_1	14.02	4.10	141	12.58	2.30	576	14.56	2.66	48
VS	56.47	0.45	135	56.61	0.43	495	56.55	0.39	288
K_2	14.79	0.72	97	15.04	0.74	354	14.38	0.71	290
	1958 year-class								
Total length	25.67			24.47			26.81		
l_1	15.76	5.60	800	12.73	5.29	1 632	16.38	4.50	16
VS	56.38	0.44	1 066	56.64	0.42	2 843	56.44	0.48	95
K_2	14.79	0.74	389	14.88	0.71	1 243	14.49	0.74	91
	1959 year-class								
Total length	26.07			24.91			26.00		
l_1	15.53	4.44	100	13.75	3.28	622	14.78	5.62	36
VS	56.34	0.48	103	56.45	0.53	933	56.37	0.50	157
K_2	14.54	0.67	102	14.95	0.72	751	14.16	0.54	152
	1960 year-class								
Total length	26.48			25.44			26.29		
l_1	15.94	3.63	1 510	12.79	3.09	1 042	16.49	3.73	478
VS	56.39	0.42	1 634	56.47	0.50	994	56.42	0.44	952
K_2	14.78	0.57	211	15.14	0.71	383	14.24	0.59	924

Using the data of 3-year-old fish submitted as Pure Stocks, l_1/total length regressions were calculated for the 1957, 1958, 1959 and 1960 year-classes; these are shown in Figure C I 2. There are no significant differences between the regressions of the three stocks for the 1957 and 1958 year-classes. The regression for the Bank stock of the 1959 year-class is significantly different from those of the Downs and Buchan stocks. This is caused by 17 anomalous values out of a total of 99; the omission of these values produces a regression similar to those of the Downs and Buchan stocks. Similarly, in the case of the Downs herring of the 1960 year-class the regression is biased downwards by the inclusion of a relatively small number of fish with low l_1 values but high total length.

From this analysis it would seem justifiable to assume that a single l_1/ total length regression can be used for standardizing total lengths to l_1 for any particular year-class. Examining the regressions in Figure C I 2 it is seen that the three stocks overlap in their distributions of l_1 and total length; the differences between stocks could be simply in the centres of their distributions. The following are the regression equations from the data shown in Figure C I 2:

1957 year-class $l_1 = 1.0668\ L - 12.8969$

1958 year-class $l_1 = 1.7054\ L - 28.5960$

1959 year-class $l_1 = 0.9437\ L - 9.5368$

1960 year-class $l_1 = 1.1113\ L - 13.8450$

To standardize length to l_1 the value of l_1 calculated from the regression equation is taken, and this calculated l_1 is referred to as the standardized l_1. The variance of the distribution of the standardized l_1 is not the same as that of the original l_1 distribution but may be expressed as $b^2 \times S^2$, where b is the slope of the regression $y = a + bx$, and S^2 is the variance of the total length, x. Neither the values of the standardized l_1 nor the variances of their distributions about the means are directly comparable with those of the observed l_1 values.

The length data of 3-year-old herring have been obtained by combining length measurements made by the English, German, Dutch and Scottish laboratories. For this purpose weekly length compositions per stock per year-class were transformed to per mille compositions and averaged (Table C I 2). The means and variances of the total length distributions were calculated, and these mean lengths used to calculate the standardized l_1s from the l_1/total length regressions. The standardized l_1 variances have been derived as described above.

The discriminant function analysis works on the assumption that the variance of a character is the same in all three stocks. Therefore, for each year-class a common variance for the standard l_1, VS and K_2 has been computed as the simple mean of the variances found in the three spawning stocks. Table C I 3 shows, for each year-class, the mean standardized l_1, VS and K_2 values of the three Pure Stocks and the common variance for each character. The l_1 values and the variances used in the discriminant analysis are given in Table C II 8; these are different from the values in Table C I 3 and their derivation is described in section C II.

Table C I 2 Length distributions (‰) and means of 3-year-old Buchan, Bank and Downs spawners of the 1957, 1958, 1959 and 1960 year-classes, obtained by combining data from England, Germany, Scotland and Netherlands

	Length (cm)											Mean length (cm)
	20	21	22	23	24	25	26	27	28	29	30	
	1957 year-class											
Downs	1.7	38.1	183.8	372.8	301.3	97.4	4.9					23.75
Bank		1.0	29.3	198.1	358.5	273.8	116.0	23.3				24.82
Buchan			2.9	24.0	152.0	280.8	244.6	291.6	4.1			26.13
	1958 year-class											
Downs	1.5	19.2	117.6	315.5	314.8	171.9	48.9	8.8	1.6			24.19
Bank		0.1	9.5	74.7	203.5	313.5	324.4	67.3	6.8	0.2		25.60
Buchan					50.9	126.9	329.6	247.7	204.9	40.0		27.05
	1959 year-class											
Downs	0.1	1.5	19.6	136.9	336.6	348.0	142.4	14.0	0.9			25.00
Bank				13.0	223.3	310.7	278.9	122.7	48.8	2.6		25.93
Buchan				14.8	33.7	208.8	388.7	231.6	111.8	7.6	3.0	26.67
	1960 year-class											
Downs	0.1	0.5	9.6	145.6	322.7	359.7	151.3	38.7	2.6	0.3		25.10
Bank		0.4	7.8	34.9	131.9	256.5	321.9	195.9	45.8	5.1		26.10
Buchan			1.1	28.1	116.9	206.7	296.6	250.5	93.3	2.2	3.4	26.43

Table C I 3 Means and variances of Pure Stock parameters

Stock	Mean	Year-class			
		1957	1958	1959	1960
Bank	l_1	13.52	14.97	14.89	15.19
	VS	56.47	56.38	56.34	56.39
	K_2	14.79	14.79	14.54	14.78
Downs	l_1	12.38	12.57	14.01	14.10
	VS	56.61	56.64	56.45	56.47
	K_2	15.04	14.88	14.95	15.14
Buchan	l_1	14.93	17.45	15.59	15.54
	VS	56.55	56.44	56.37	56.42
	K_2	14.38	14.49	14.16	14.24
Variances	l_1	1.34	3.93	1.09	2.24
	VS	0.42	0.44	0.50	0.45
	K_2	0.70	0.68	0.69	0.66

C II $\underline{l_1}$

In the preliminary analysis of the discriminant powers of the available characters, l_1, VS and K_2, Andersen showed that l_1 was of greatest value (ICES 1963, mimeo). It is unfortunate that this character, which is the most powerful of the three, is also the most difficult to use. The difficulties arise both from the limited numbers of fish with scales and also from the basic statistical requirements of the discriminant analysis.

The underlying hypothesis in the use of the discriminant analysis is that the true values of the Pure Stock population parameters are known. Thus for the Downs, Bank and Buchan stocks we need to have the means of the true l_1 distributions for each stock and it must be assumed that their variances are identical. In practice we do not know these values precisely, we only have better or worse estimates dependent on sample sizes and the way these were obtained. The form of the l_1 distribution obtained is dependent upon the true l_1 distribution of the population, modified due to the sampling errors.

The distributions may be biased for a great number of reasons; a few of the more serious ones will be discussed here. It is well known that l_1 and total length are highly correlated within an age group. Thus, dependent upon the type of gear used for capture, it is often the case that fish of particular length groups have few scales suitable for l_1 determination. This would result in a biased l_1 distribution. In the material available few fish had scales, and the problem of the representativeness of the fish used in the analysis will be discussed fully later.

An analysis of the nature of l_1 distributions has been made by Burd and Parnell (1969, in press). They showed that a considerable part of the variance in l_1 was ascribable to the part of the fish from which the scale had been taken.

On any fish, scales from the mid line forward of the dorsal fin gave minimal estimates of l_1 when proportioned linearly in the usual manner. Any change from this general area resulted in increasing values of l_1. In a population of fish from which scales have not been taken systematically from this area, the l_1 distribution is likely to have too great a mean and variance. The bias will always be positive.

When data are being combined from different sources, as in this case, national differences may exist in the manner in which the scales are read and measured and interpreted. Andersen reported the results of a comparison on a standard set of scales made by the countries involved in the surveys. Systematic differences were found, which indicated methodological differences between countries. At the time when the data from the surveys were worked up it was found that in one of the laboratories the equipment used had been malfunctioning. From another laboratory it was later found that the data had been reported in 0.25 cm groupings.

The l_1 distributions for the young herring are thus likely to be biased by these types of errors. The bias would be likely to give values of the mean and variance which are too high.

In the Pure Stock characters much the same errors may arise. However, the effects of the errors may be different between the Pure Stocks and the young herring. The requirement that the variances of the Pure Stocks within the mixed young herring should be the same might thus not be valid.

When the data from the young herring were separated into year-classes it was found that very few fish in the research vessel samples had scales. A discriminant analysis based on these few fish, with the likely errors involved in the means and variances, would be unlikely to provide useful results. The numbers of fish in each survey in maturity stages I and II for which l_1 values were available are given in the following table.

Table C II 1 Numbers of young nerring having l_1 values

Year-class	Region	1960				1961			
		Spring		Autumn		Spring		Autumn	
		Maturity stages							
		I	II	I	II	I	II	I	II
1957	W	279	99	21	18	1	25	0	0
	NE	21	7	0	0	6	1	0	0
	SE	186	35	8	3	48	28	0	0
1958	W	155	2	100	40	45	104	3	5
	NE	0	0	2	0	63	3	2	5
	SE	191	0	946	4	328	100	0	3
1959	W			0	0	44	3	19	9
	NE			0	0	9	0	16	0
	SE			0	0	55	1	86	2
1960	all							0	0

In order to utilize a length character in the discriminant analysis it was decided to calculate an l_1 value for each fish measured, using the l_1/total length regressions obtained from the above data. This is equivalent to using total length as the discriminant character but standardized to the length at the first winter ring. The variance of l_1 about the regression line is considered to include the variances due to biases in l_1 determination.

Separate l_1/total length regressions were calculated for the year-classes in which there were sufficient data in maturity stages I and II. These are shown in Figure C II 1. In no cases were significant differences found between the different maturity stages over the observed range of l_1 and total length.

The maturity data were then combined and examined to see if there might be differences in the regressions for different areas in the North Sea. The regressions are shown in Figure C II 2. There were no significant differences between areas within surveys for the separate year-classes.

In view of the lack of differences within a year-class either between maturity stages or between areas, all the data were combined to give a seasonal regression for each year-class for the whole North Sea. The equations of the l_1/total length regressions are given in Table C II 2. The 1959 and 1960 year-classes in the autumns of 1960 and 1961 respectively are 0-group and their total lengths have been taken as l_1.

Table C II 2 Equations of l_1/total length regressions

Year-class	Season	Survey 1960	Survey 1961
1957	Spring	$y = 1.00727x - 7.82386$	$y = 0.407918x + 3.45470$
	Autumn	$y = 0.728521x - 5.13669$	
1958	Spring	$y = 0.831447x + 2.54814$	$y = 1.10007x - 10.2934$
	Autumn	$y = 1.14935x - 9.83020$	$y = 1.56572x - 24.9316$
1959	Spring		$y = 1.03449x - 1.94240$
	Autumn		$y = 0.974280x - 6.39746$

For each fish in the biological samples of these year-classes standard l_1 values have been derived and mean standard l_1s and their variances have been calculated for each statistical rectangle for each year-class. Table C II 3 gives a summary of the standard l_1s, grouped by regions; these indicate that within a year-class there is a tendency for the fish with the smallest l_1s to be in the south-eastern region at any time. In most cases this is also true of the variances.

Table C II 3 Mean, variance and number of observations of standard l_1 values in centimetres by year-class, survey and area

Year-class	Survey		Region: West	Region: North-east	Region: South-east	Mean variance
1957	Spring 1960	$\bar{x}$	13.35	13.41	12.24	2.63
		s^2	2.69	3.22	2.0	
		n	1 217	585	685	
	Autumn 1960	$\bar{x}$	11.61	14.82	11.13	1.60
		s^2	1.67	0	1.25	
		n	165	1	25	
	Spring 1961	$\bar{x}$	13.40	13.46	13.08	0.28
		s^2	0.24	0.70	0.25	
		n	92	17	96	
1958	Spring 1960	$\bar{x}$	16.48	16.21	14.49	2.88
		s^2	2.28	2.71	3.04	
		n	495	1 079	3 035	
	Autumn 1960	$\bar{x}$	15.61	14.84	11.64	3.15
		s^2	2.25	7.87	3.33	
		n	386	6	1 803	
	Spring 1961	$\bar{x}$	13.72	12.27	12.20	3.99
		s^2	3.78	5.61	3.62	
		n	557	311	995	
	Autumn 1961	$\bar{x}$	13.40	12.22	10.41	5.16
		s^2	4.48	14.54	3.52	
		n	24	4	13	
1959	Autumn 1960	$\bar{x}$	14.56	0	14.30	1.52
		s^2	1.84	0	1.37	
		n	152	0	299	
	Spring 1961	$\bar{x}$	14.85	15.08	14.98	2.92
		s^2	2.81	2.94	1.91	
		n	351	441	608	
	Autumn 1961	$\bar{x}$	14.02	13.93	13.93	2.53
		s^2	2.79	1.58	2.35	
		n	206	16	171	

During the course of the surveys, at each trawling station some 50-100 young herring were taken for the biological sampling. In addition the remainder of the catch was measured, but these data have not been used in relation to the fish from the biological sample; it has been assumed that the latter are representative of the fish caught.

The fish used for the discriminant analysis were those fish which had either VS or VS and K_2. Thus the data on which the discriminant functions were used are themselves a sub-sample of the biological sample. Of the three discriminant analyses made, l_1/VS/K_2, VS/K_2 and l_1/VS, the latter is based on most observations, because fewer K_2 values than VS were counted.

Table C II 4 Comparison of mean lengths of samples and fish used in the analysis of l_1/VS/K_2

Year-class	Season	Mean lengths (cm)	
		All fish	Fish used
1957	Spring 1960	20.55	20.79
	Autumn 1960	23.92	24.12
1958	Spring 1960	14.51	15.31
	Autumn 1960	18.91	19.34
	Spring 1961	20.30	21.13
	Autumn 1961	24.50	24.59
1959	Autumn 1960	14.29	14.58
	Spring 1961	16.36	16.36
	Autumn 1961	21.74	21.13
1960	Autumn 1961	14.39	14.47

There is a tendency for the fish on which the analysis was made to be slightly larger. The standard l_1 values of the young herring analysed will be somewhat higher than that of all the fish in the biological sample. The effects of such a bias would be to introduce an underestimate of the Downs proportion of the mixed stock.

A further bias due to l_1 might exist in the stock separation. The value of l_1 for the Pure Stock characters is dependent upon adequate sampling of the adult spawning stock. In the case of the 1958 year-class of both Bank and Buchan herring, it has been suggested that the mean lengths of the 3-year-old recruits are biased. Certainly for the Buchan stock the standard l_1 so derived is much higher than would be expected from examination of the differences between stocks on a long-term basis. The effect of such a bias would be to underestimate the proportions of Bank and Buchan stock in the young herring.

In the case of the 1958 year-class the overall effects of these two biases might be to eliminate any underestimation, since the two are working in opposite directions. In the three other year-classes the effects of selection of the fish used might be to underestimate the Downs component by a slight amount in the total North Sea analysis.

When the differences between fish used and all fish measured are examined on an area basis, it is seen that any bias might be restricted to particular parts of the North Sea. Table C II 5 summarizes the area differences. In the case of the 1957 year-class there is a tendency for the fish analysed to be larger in the east

Table C II 5 Comparison of the total lengths (cm) of the fish used for l_1/VS/K_2 discrimination with those of all fish measured, by areas

1957 year-class

SPRING 1960

West

21.37 = all fish (n = 11 886)
21.09 fish used

East

19.20 = all fish (n = 7 208)
20.71 fish used

AUTUMN 1960

West

24.30 = all fish (n = 4 079)
24.11 fish used

East

23.06 = all fish (n = 1 797)
24.14 fish used

1958 year-class

SPRING 1960

West

17.04 = all fish (n = 3 414)
16.67 fish used

North-east

16.58 = all fish (n = 6 114)
16.45 fish used

South-east

14.27 = all fish (n = 88 126)
14.58 fish used

AUTUMN 1960

West

22.35 = all fish (n = 1 423
22.16 fish used

North-east

22.9 = all fish (n = 10)

South-east

18.79 = all fish (n = 39 735)
18.78 fish used

SPRING 1961

West

21.94 = all fish (n = 4 175)
21.85 fish used

North-east

18.96 = all fish (3 221)
or excluding P 13
20.90 = all fish (1 061)
20.55 fish used

South-east

19.94 = all fish (6 854)
or excluding M 07
20.75 = all fish (4 145)
20.82 fish used

AUTUMN 1961

West

24.86 = all fish (n = 10 631)
24.90 fish used

North-east

No fish

South-east

24.50 = all fish (n = 1 986)
24.23 fish used

1959 year-class

AUTUMN 1960

West

13.99 = all fish (n = 1 079)
14.60 fish used

East

14.42 = all fish (n = 2 632)
14.56 fish used

SPRING 1961

West

16.23 = all fish (n = 3 747)
16.52 fish used

East

16.40 = all fish (n = 24 243)
16.29 fish used

AUTUMN 1961

West

21.72 = all fish (n = 1 683)
21.18 fish used

East

21.79 = all fish (n = 2 153)
21.08 fish used

than all fish in the biological sample and smaller in the west. A similar feature is seen in the 1959 year-class. In the more detailed breakdown by areas given for the 1958 year-class the same tendencies exist.

The values of the Pure Stock l_1 parameters given in Table C I 1 and first used in the preliminary discriminant analyses in 1965 were the observed means and variances of the relatively few spawning fish available. It was clear from the results of the discriminant analysis that the mean values and the variances were far too great. Indeed some Pure Stock mean l_1 values were outside those observed in the young herring standard l_1 population.

As has been noted previously it was decided that the Pure Stock mean l_1 ought to be used as a standardized value. This introduced a great weight of adult herring length data and increased the reliability of the values used. However, it was found that these values and their variances also gave results in the discriminant analysis which proved untenable. It appeared that the requisite underlying hypothesis of the equality of variances and the means of the stock components in the young herring and in the Pure Stocks was unlikely to be true. It was considered that the sampling errors on the Pure Stocks were still probably too large to satisfy the basic statistical hypotheses for the discriminant analysis.

In order to try to resolve this difficulty the following hypothesis was set up. It was assumed that within the young herring data from the North Sea, all three stocks were present and that their variances were identical. Within the Pure Stocks, though the mean l_1 for each stock was biased, it was assumed that the differences between the mean l_1 of Bank and Downs, Downs and Buchan and Bank and Buchan were identical to the differences in these stocks in the young herring. The discriminant analysis was then used in an iterative manner to obtain estimates of the stock means and variances which fulfilled the underlying hypothesis of identity of these parameters in both Pure Stock and juvenile stock.

In Table C II 6 the iterative technique is illustrated with the working of the 1957 year-class in the spring 1960 survey. In order to obtain the first set of discriminant functions the Pure Stock standard l_1 values are those obtained from the spawning fish. The variance used is that for the standard l_1 of the total fish sampled. Using the limits for stock separation as indicated in columns (a) to (f) inclusive, the stock separation is given in (g), (h) and (j); these, however, have to be corrected for misclassification, which results in the adjusted numbers in (k), (l) and (m).

Assuming that these frequencies represent the mixture of the three Pure Stocks among the young herring, then

$$\hat{M}_s = \frac{N_B \times M'_B + N_D \times M'_D + N_{Bu} \times M'_{Bu}}{N_B + N_D + N_{Bu}},$$

where $\hat{M}_s$ is the calculated mean l_1 of the total analysed fish. M'_B, M'_D and M'_{Bu} are the estimated mean l_1s of Bank stock, Downs stock and Buchan stock respectively, and N_B, N_D and N_{Bu} (columns (k), (l), (m)) are the proportions of these stocks in the juvenile herring. In the first iteration

$$\hat{M}_s \neq M_s,$$

Table C II 6 Iterative procedure for determining Δl_1 and l_1 variances

Spring 1960 survey - 1957 year-class - whole area

Initial mean values and variances (Pure Stock)

Character	Bank	Downs	Buchan	Variance
l_1(M') (cm)	13.5235	12.3817	14.9267	?
VS	56.4700	56.6100	56.5500	0.4200
K_2	14.7900	15.0400	14.3800	0.7000

(VS and K_2 values remain unchanged throughout)

Characters used: l_1/VS/K_2 Mean l_1 of total sampled fish (M_s) = 13.1516 cm Variance = 3.1574

Iteration	Stock centres						Stocks as classified			Adjusted numbers			Δl_1	Adjusted l_1 variance
	Bank		Downs		Buchan		Bank	Downs	Buchan	Bank	Downs	Buchan		
	X (a)	Y (b)	X (c)	Y (d)	X (e)	Y (f)	(g)	(h)	(j)	(k)	(l)	(m)	(n)	(p)
1	-19.2151	11.1116	-19.7640	12.2877	-18.5879	9.60547	346	630	408	347.0	737.0	300.0	-0.06812	2.12875
2	-16.8884	5.92559	-17.6368	7.54637	-16.0160	3.87287	361	625	398	250.5	771.6	362.0	-0.10243	1.98830
3	-16.3983	4.83322	-17.1900	6.55042	-15.4728	2.66200	358	620	406	205.2	782.3	396.5	-0.12847	1.91574
4	-16.1219	4.21713	-16.9384	5.98968	-15.1658	1.97788	356	618	410	180.7	789.2	414.1	-0.14070	1.87840
5	-15.9707	3.87996	-16.8007	5.68267	-14.9979	1.60365	356	614	414	176.0	784.4	423.6	-0.15422	1.86334
6	-15.9132	3.75192	-16.7488	5.56713	-14.9336	1.46025	359	611	414	193.2	772.1	418.7	-0.15940	1.87967
7	-15.9875	3.91756	-16.8171	5.71923	-15.0154	1.64255	360	610	414	202.4	766.7	415.0	-0.16017	1.88974
8	-16.0312	4.01500	-16.8571	5.80843	-15.0637	1.75010	360	610	414	204.0	766.0	414.0	-0.15972	1.89197

where M_s is the observed mean l_1 of the young herring. The difference between these two identities is Δl_1 (n). This Δl_1 is subtracted from the Pure Stock means to give the new values for the next iteration. The variance is adjusted in the following manner.

The total variance S^2 of the standard l_1 for all fish in a year-class in a survey may be expressed as

$$S^2 = \sigma^2 + \frac{N_D (M_s - M'_D)^2 + N_B (M_s - M'_B)^2 + N_{Bu} (M_s - M'_{Bu})^2}{N_D + N_B + N_{Bu}}$$

where σ^2 is the common variance of the true l_1 distributions. M_s is the observed mean of all standard l_1s in the survey, and M'_D, M'_B, M'_{Bu}, N_D, N_B, N_{Bu} are the respective estimates of Pure Stock means and their abundances in the young herring. The iterations proceed until the difference between the last two estimates of l_1 is less than 0.01. Table C II 7 gives the values of Δl_1 and the Pure Stock variances obtained from the total North Sea data. It is supposed that the Pure Stock characters are diagnostic of the year-class throughout its life. Then the values of Δl_1 and the variance obtained for each year-class from the different surveys may be taken as estimates of the stock means. For the final analysis by the discriminant function weighted means of the Δl_1 and variance have been used. The changes to the value of the l_1 stock character used, caused by these various manipulations, are summarized in Table C II 8.

Table C II 7 Derived Δl_1 and Pure Stock variance

Survey			Year-class			
			1957	1958	1959	1960
1960	Spring	Δl_1	-0.1597	-0.0320		
		σ^2	1.8920	1.2139		
	Autumn	Δl_1	-0.1210	-1.3062	-0.2346	
		σ^2	0.9751	1.7479	0.9694	
1961	Spring	Δl_1		-0.9982	-0.0952	
		σ^2		1.8211	2.0889	
	Autumn	Δl_1		+0.1399	-0.9108	-0.9590
		σ^2		2.6220	1.9009	1.7213
Weighted mean		Δl_1	-0.1426	-0.4609	-0.2925	-0.9590
		σ^2	1.4862	1.6919	2.0434	1.7213

Table C II 8 Mean l_1 and standardized l_1 values (cm) for year-classes

Year-class	Stock	Observed mean l_1	Standard mean l_1	Iterated mean l_1
1957	Bank	14.02	13.52	13.38
	Downs	12.58	12.38	12.24
	Buchan	14.56	14.93	14.79
	Variance	2.65	1.34	1.49
1958	Bank	15.76	14.97	14.51
	Downs	12.73	12.57	12.11
	Buchan	16.39	17.45	16.99
	Variance	5.39	3.93	1.69
1959	Bank	15.53	14.89	14.60
	Downs	13.75	14.01	13.71
	Buchan	14.78	15.59	15.30
	Variance	3.54	1.09	2.04
1960	Bank	16.19	15.19	14.23
	Downs	13.04	14.10	13.15
	Buchan	16.74	15.54	14.58
	Variance	4.03	2.24	1.72

The changes between the observed and the standard l_1 means reflect the differences in the total length distributions of the l_1-sampled fish given in Table C I 1 and those of the 3-year-old Pure Stocks given in Table C I 2. The iterated values are the values of Pure Stock l_1 and variances which have been used in the final discriminant function.

C III K_2 and VS values

Average values of keeled scales and vertebrae are given by areas in Table C III 1 for the young herring. The area values are further set out in Figure C III 1, together with the Pure Stock values used in the discriminant analysis.

Generally speaking, the values of these characters for the 1957 and 1958 year-classes are not incompatible with the postulate that the immature herring are mixtures of the three main North Sea stocks.

In the 1959 year-class the K_2 and VS values for the young herring are rather different from what might be expected from the Pure Stock values (Figure C III 1). Three explanations may be offered:

1. The values of K_2 and VS in the young herring are wrong. This is most unlikely since these fish were collected in the same samples as those for the 1957 and 1958 year-classes and were counted by the same people (section A V (iv)).

Table C III 1 K_2 and VS by areas

			West	North-east	South-east
	(i)	1957 year-class			
Spring 1960	VS	mean	56.549	56.478	56.519
		number	990	561	624
		variance	0.515	0.489	0.507
	K_2	mean	14.788	14.536	14.872
		number	387	491	462
		variance	1.038	0.813	0.723
Autumn 1960	VS	mean	56.471	56.500	56.515
		number	1 107	12	270
		variance	0.521	0.273	0.466
	K_2	mean	14.715	14.250	14.873
		number	853	12	267
		variance	0.901	0.568	0.736
Spring 1961	VS	mean	56.543	56.510	56.485
		number	221	102	97
		variance	0.431	0.609	0.586
	K_2	mean	14.672	14.398	15.024
		number	204	108	84
		variance	0.645	0.616	0.891
	(ii)	1958 year-class			
Spring 1960	VS	mean	56.501	56.564	56.518
		number	389	1 060	2 772
		variance	0.483	0.543	0.467
	K_2	mean	14.922	14.377	14.838
		number	204	832	1 705
		variance	1.235	0.707	0.771
Autumn 1960	VS	mean	56.413	56.500	56.536
		number	351	4	1 734
		variance	0.569	0.333	0.453
	K_2	mean	14.592	14.667	14.865
		number	311	3	1 709
		variance	0.894	2.333	0.681
Spring 1961	VS	mean	56.536	56.478	56.487
		number	515	295	904
		variance	0.401	0.509	0.496
	K_2	mean	14.700	14.753	14.828
		number	443	292	454
		variance	0.812	0.771	0.593
Autumn 1961	VS	mean	56.576	56.800	56.464
		number	1 289	5	973
		variance	0.432	1.200	0.461
	K_2	mean	14.772	13.600	14.840
		number	1 095	5	946
		variance	0.644	0.800	0.689

Table C III 1 continued

			West	North-east	South-east
	(iii)	1959 year-class			
Autumn 1960	VS	mean	56.599	-	56.510
		number	142	0	249
		variance	0.398	-	0.509
	K_2	mean	14.210	-	14.495
		number	143	0	216
		variance	0.618	-	0.660
Spring 1961	VS	mean	56.459	56.556	56.484
		number	362	459	640
		variance	0.448	0.444	0.501
	K_2	mean	14.022	14.497	14.658
		number	322	463	424
		variance	0.657	0.705	0.765
Autumn 1961	VS	mean	56.441	56.500	56.447
		number	202	16	161
		variance	0.586	1.200	0.486
	K_2	mean	14.289	14.500	14.569
		number	166	16	167
		variance	0.704	0.533	0.777
	(iv)	1960 year-class			
Autumn 1961	VS	mean	56.466	56.441	56.402
		number	58	238	1 831
		variance	0.429	0.442	0.412
	K_2	mean	14.146	14.344	14.466
		number	48	215	1 757
		variance	0.510	0.498	0.616

2. The Pure Stock characters are not representative. In particular this could be true of the Buchan spawners. The difficulty of sampling Buchan spawners is a long-standing one. Table C III 2 shows the derivation of the mean values used for the Buchan Pure Stock characters. The fish in maturity stages VI and VII, which have been used, were captured not only on the Buchan grounds proper, off Aberdeen, but also from Clythness in the northern Moray Firth and at Shetland. It is seen that in the 1959 year-class almost all the fish are derived from the Clythness spawning ground, because there was almost no fishing on the Aberdeen grounds by Scottish vessels in 1962. However, the fish spawning in the Clythness area are part of the total spawning stock of the north-western North Sea. The relative differences in annual sampling rate are considered relative to the importance of the catches of the spawning fisheries landed in Scotland from these areas. The mean values for the 1959 year-class are not considered to be any less representative of the total spawning fishery than in the case of the other year-classes.

Table C III 2 Derivation of the Buchan Pure Stock characters for the 1957, 1958 and 1959 year-classes; figures in parentheses are the numbers of fish

Year-class		Pure Stock means	Buchan area	Clythness area	Shetland area
1957	VS	56.55	56.60 (114)	56.53 (101)	56.48 (85)
	K_2	14.38	14.47 (110)	14.37 (101)	14.25 (87)
1958	VS	56.44	56.63 (43)	56.34 (47)	56.70 (20)
	K_2	14.49	14.55 (42)	14.48 (42)	14.37 (19)
1959	VS	56.37	56.44 (9)	56.32 (143)	56.64 (22)
	K_2	14.16	14.22 (9)	14.15 (141)	13.95 (20)

3. In a year-class in which the relative strengths of the North Sea stocks were weak, immature fish from other stocks could then have an increased importance in the North Sea juvenile stock. This could only be detected if the Pure Stock characters of such a component were very different from those used in the analysis, for example spring spawners (section A VI).

Saville (1969, in press) has shown that the immatures of the Moray Firth are derived from the Minch autumn spawning stock, and there is no reason why the area inhabited by these immatures may not be more extensive. Table C III 3 gives the values of VS and K_2 for the Minch autumn spawning herring. Plotting the Pure Stock characters of the Minch spawning stock in the VS/K_2 diagram (Figure C III 1), it is obvious that the mean values for the immature fish are much better covered when the Minch characters are included. This suggests that it is highly probable that a big proportion of the immature fish belonged to a spawning stock described by VS and K_2 characters not too different from the Minch stock. An even better fit is obtained using the characters of the Shetland spawners. However, the numbers of fish from which these are derived are low.

For comparison, the values of these Pure Stock characters for the 1957 and 1958 year-classes have also been plotted in Figure C III 1. Despite the low numbers in the 1958 year-class these Pure Stock values consistently show a close association with the loci of the young herring.

Table C III 3 Pure Stock characters of the Minch autumn spawning herring

Mean	Year-class			
	1957	1958	1959	1960
VS	56.52	56.59	56.53	56.50
K_2	14.17	14.27	14.19	14.20

Figure C III 2 shows the plots of l_1 on K_2 for the young herring and the Pure Stocks. The l_1 values for the young herring are the standard l_1 values used in the discriminant analysis; the l_1 values of the Bank, Downs and Buchan stocks are the iterated l_1 values used in the linear functions used for discrimination. A discriminant function using l_1 and K_2 gave statistically acceptable allocations of the young herring between the Pure Stocks described by the mean values shown in Figure C III 2.

Also shown in Figure C III 2 are the Minch spawning stock loci. An l_1/ total length regression for the 1959 year-class was calculated using data supplied by Mr A. Saville with some additional data from the Lowestoft laboratory. The regression equation obtained, $l_1 = 0.4917L - 0.1722$, may be compared with those for the North Sea stocks in section C I, page 25. It is seen that the slope for the Minch spawners is about half that for the North Sea spawners. In the absence of data for other year-classes this regression has been used to calculate standard l_1 values for each year-class of Minch spawners; the results are given in Table C III 4.

Table C III 4 Mean lengths of 3-year-old Minch autumn spawners and derived standard l_1 values

Year-class	Mean length (cm)	Standard l_1 (cm)
1957	23.92	11.59
1958	24.03	11.64
1959	24.88	12.35
1960	25.24	12.24

In plotting these standard l_1 values in Figure C III 2 they have been reduced by the iterative correction for the 1959 year-class. In the 1959 and 1960 year-classes the close similarity in mean K_2 of the Minch and Buchan estimates is noteworthy.

The l_1 characters in all year-classes are, however, very different. It is seen that the young herring have l_1 values intermediate between these two extremes.

It is concluded that the hypothesis that the juvenile herring stock was composed only of fish described by their spawning meristic characters and designated as Bank, Downs and Buchan stock was erroneous for the 1959 year-class. The l_1/K_2 plot might suggest a similar situation in the 1960 year-class, but it must be remembered that these fish were sampled only as 0-group in autumn 1961.

C IV Otolith types

In addition to the differences in otoliths between spring- and autumn-spawned herring discussed in section A VI it has been shown that within the autumn-spawned component there are also differences in otolith characteristics. The most useful of these, and the one most studied, has been the nature of the first winter ring. Parrish and Sharman (1958) have shown that autumn-spawned otoliths can be divided into two groups - those with a wide or diffuse first winter ring and those with a thin, sharp, well-defined first winter ring. Otoliths in the first category have a small first growth zone, whilst those with a "narrow" first winter ring have a wide range of first growth zone measurements, but in general a high modal first growth zone. Subsequent investigation of this character has shown that in general Buchan spawners have a high proportion of "narrow" type otoliths, and Downs spawners a high proportion of "wide" type otoliths, while Bank spawners are intermediate between the other two (Parrish and Sharman 1959; Das, Postuma and Zijlstra 1959; Raitt 1961). This would appear to be a character which might be used in checking whether the allocation to spawning groups by the discriminant function analysis is compatible with evidence from an independent source.

Unfortunately the usefulness of this character is limited in several respects: (a) the distinction between a "wide" and "narrow" ring is largely a subjective judgement which can be made with some constancy within a laboratory but is difficult to standardize between workers in different countries; (b) the proportion of "wide" and "narrow" ring types within a spawning group can vary considerably between year-classes; (c) the distinction between ring types can only be made after some growth has been added to the otolith after the first winter ring has been formed. Thus in the present material this character is first available, for a year-class, for fish caught in the autumn survey as 1-group.

As regards (a) Parrish and Sharman (1958) appreciated that it was not possible to allocate all otoliths either to the narrow or wide categories, and they accordingly had a third category of doubtful otoliths which they designated "?". This three-way classification has since been adopted by all countries doing otolith classification, but even so there seems to be considerable variation between laboratories as to where the dividing lines are drawn. That the allocation of herring to otolith types is not consistently applied between countries has already been shown in the Report of the Working Group on Methods used in North Sea Herring Investigations (Anon 1962). The data given there are inadequate to measure the discrepancies between countries but suggest that Denmark and Germany allocate considerably more herring to the "wide" type than do Scotland, England and the Netherlands. This conclusion is supported by the data under consideration here. For spawners of the Bank and Downs stocks material was available for calculating the proportions of otolith types within the Pure Stocks from England, Germany and the Netherlands. Only for the 1958 year-class was the German material sufficiently numerous to give a valid otolith type allocation for that country; for that year-class for the Downs stock the percentage allocations were:

	Narrow	Wide	?
England	50.8	47.5	1.6
Netherlands	49.3	49.4	1.3
Germany	23.8	72.2	4.0

These figures would again suggest that Germany allocated many more herring to the "wide" type than do England and the Netherlands, whereas the results from the latter two countries were reasonably compatible.

That some countries are allocating more fish to the "wide" type than others is also suggested by the allocation to otolith types within a year-class over all the fish sampled in a survey. These are shown in Table C IV 1, together with the Pure Stock otolith allocations. In almost all cases the proportion of "wide" otolith types within a year-class in the survey data is higher than in any of the Pure Stocks of which the mixed population of young fish is composed.

Table C IV 1 Comparison of frequencies of "wide" and "narrow" otolith types in Pure Stocks and juvenile herring

Year-class	Data source	Survey	Narrow	Wide	?
1957	Survey data	Spring 1960	45.92	46.12	7.87
		Autumn 1960	39.57	50.21	10.22
		Spring 1961	55.16	35.01	9.83
	Pure Stock data	Bank	65.94	24.77	9.29
		Downs	44.66	49.22	6.12
		Buchan	89.08	7.56	3.36
1958	Survey data	Autumn 1960	32.30	61.90	5.80
		Spring 1961	36.14	57.84	6.02
		Autumn 1961	37.07	58.05	4.88
	Pure Stock data	Bank	66.23	31.80	1.97
		Downs	49.36	49.40	1.24
		Buchan	88.64	9.85	1.51
1959	Survey data	Autumn 1961	70.95	20.39	8.66
	Pure Stock data	Bank	90.24	8.94	0.81
		Downs	73.25	24.78	1.97
		Buchan	78.23	19.05	2.72

As the Pure Stock allocations to otolith type are based solely on the analysis of English, Netherlands and Scottish data, whereas a high proportion of the survey material comes from Danish and German otolith data, this would suggest that Germany and Denmark are allocating a higher proportion of fish to the "wide" type than the other countries.

In an attempt to get a quantitative measure of the discrepancies between countries the otolith typing from the international surveys was examined in all cases where two countries sampled the same statistical square on a single survey. In this analysis of the data it was accepted that England, Netherlands and Scotland were reasonably consistent in their allocation to otolith type, as is suggested by the data reported in Anon (1962), and by the Pure Stock data for otoliths, at least as far as England and the Netherlands are concerned. In this case the objective is to obtain correction factors to bring German and Danish otolith typing into agreement with that of the other three countries. So all the English, Netherlands and Scottish otolith data from squares where a ship of one of these countries had fished in the same square on the same survey and on the same year-class as Denmark and Germany were grouped, giving the results shown in Table C IV 2.

Table C IV 2 Comparison of otolith type allocation by Denmark and Germany and the other countries

Type						Type					
Narrow		Wide		?		Narrow		Wide		?	
%	No.	%	No.	%	No.	%	No.	%	No.	%	No.
England, Scotland and Netherlands						Germany					
63.3	198	33.0	100	1.7	5	42.0	342	44.9	366	13.1	107
England, Scotland and Netherlands						Denmark					
35.7	66	58.4	108	5.9	11	10.7	32	81.7	245	7.7	23

These results also suggest that Denmark and Germany are allocating more otoliths to the "wide" and "?" categories than are the other three countries. They are not completely satisfactory for making correction factors but are the best available, and so the following correction factors based on these data have been applied to German and Danish otolith allocations: Germany "narrow" add 23.3 per cent, "wide" subtract 11.9 per cent, "?" subtract 11.4 per cent; Denmark "narrow" add 25.0 per cent, "wide" subtract 23.3 per cent, "?" subtract 1.7 per cent. These correction factors have been applied to all the otolith data analysed by Germany and Denmark. These "corrected" data were then combined with the otolith data analysed by England, the Netherlands and Scotland to give the overall percentage otolith type within sub-areas and in the total area surveyed, which are listed in Table C IV 3.

Table C IV 3   Adjusted frequencies of "wide" and "narrow" otolith types in the young herring

Year-class	Survey	North-east Percentage Narrow	North-east Percentage Wide	North-east Percentage ?	North-east No. in sample	South-east Percentage Narrow	South-east Percentage Wide	South-east Percentage ?	South-east No. in sample
1957	Spring 1960	63.2	35.9	0.9	574	40.2	55.5	4.3	634
	Autumn 1960	87.5	12.5	0.0	16	43.3	47.9	8.8	284
	Spring 1961	80.0	16.4	3.6	110	54.1	35.1	10.8	74
	Autumn 1961	25.0	25.0	50.0	4	50.9	45.5	3.6	55
1958	Autumn 1960	100.0	0.0	0.0	5	43.2	55.2	1.6	1 680
	Spring 1961	49.5	48.9	1.5	333	33.6	59.6	6.8	488
	Autumn 1961	20.0	60.0	20.0	5	45.4	53.7	0.9	979
1959	Autumn 1961	50.0	43.8	6.3	16	67.6	21.1	11.4	185

Year-class	Survey	West Percentage Narrow	West Percentage Wide	West Percentage ?	West No. in sample	Total Percentage Narrow	Total Percentage Wide	Total Percentage ?
1957	Spring 1960	72.9	22.5	4.6	845	60.1	36.4	3.5
	Autumn 1960	57.2	41.2	1.7	1 137	54.8	42.2	3.1
	Spring 1961	73.1	24.2	2.7	223	71.5	24.1	4.4
	Autumn 1961	50.6	41.0	8.4	393	50.4	41.4	8.2
1958	Autumn 1960	89.6	8.9	1.6	383	51.9	46.5	1.6
	Spring 1961	63.1	35.3	1.6	504	48.8	47.7	3.5
	Autumn 1961	67.3	32.7	0.0	1 127	57.0	42.5	0.5
1959	Autumn 1961	94.7	4.7	0.6	169	79.2	14.6	6.2

The second difficulty mentioned, concerning the variation within a stock between year-classes in the percentage of otolith types, has been met by sampling each year-class under consideration for otolith type as spawning fish. As regards Buchan spawners all of the otolith typing was done by Scotland; for the Bank and Downs stocks only English and Netherlands material was used. In this way the Pure Stock otolith characters can be taken to be consistent inter se and consistent with the corrected survey data. The distribution of otolith types of the Pure Stocks within year-classes is given in Table C IV 4.

Table C IV 4 Distribution of otolith types in the Pure Stocks

Year-class	Bank			Downs			Buchan		
	Narrow	Wide	?	Narrow	Wide	?	Narrow	Wide	?
1957	65.94	24.77	9.29	44.66	49.22	6.12	89.08	7.56	3.36
1958	66.23	31.80	1.97	49.36	49.40	1.24	88.64	9.85	1.51
1959	90.24	8.94	0.81	73.25	24.78	1.97	78.23	19.05	2.72

C V Maturity of 3-year-old herring; recruitment to the adult stock

Burd (in Burd and Cushing 1962) showed that immature virgin herring start to mature at about 21 cm, irrespective of age. This observation was made on samples from the summer feeding concentrations off the English north-east coast, i.e. during the main period of growth. As length increased, more and more herring passed into maturing stages.

The present material, collected either in March/April before the feeding period commences or in September when growth has considerably slowed down, is not well suited for the purpose of testing whether the critical length found by Burd for the North Shields herring is also valid in other areas. It does show, however, that in the 1958 year-class partial recruitment is of small importance. Table C V 1 shows the percentage maturity composition of this year-class in autumn 1960 and in spring and autumn 1961, corresponding to ages of approximately 2, 2½ and 3 years respectively; the table also shows the data for the 1957 year-class in autumn 1960, corresponding to an age of 3 years. The material is shown by the areas used for estimates of fishing power and abundance (section B IV).

As could be expected, at 2 years of age very few herring had yet reached maturity. Of the total number of fish examined in autumn 1960 and spring 1961 98.5 per cent were immature virgins. After the four months of vigorous growth separating the two surveys in 1961 only 8.1 per cent were left as immatures, and these would most likely not recruit until the following year.

In autumn 1961, nearly all the material was derived from the central North Sea and the table shows a peculiar difference between the eastern and western parts; it appears that maturation is relatively retarded in the latter area, despite the larger size of the fish there. Table C V 2 shows the length distribution of herring by maturity groups. It is evident that, size for size, maturity stage V+

is more abundant in the eastern area. It is also seen that the percentage of immatures generally increases with size in area west; in area east, the very few immatures are only found amongst the smallest fish.

Table C V 2 shows the percentage maturity composition of the 1957 year-class in autumn 1960 and the 1958 year-class in 1961, i.e. both at 3 years of age. In the 1957 year-class, immatures comprise a far bigger portion than in the 1958 year-class, which suggests that partial recruitment was of a considerable magnitude in the former. Also, in this year-class the western area shows the highest proportion of immatures and there is again no apparent relation between length and percentage of immatures. It should be mentioned that in area west maturity V+ includes a considerable number of spents, probably from the Buchan stock, and judging from the VS/K_2 discrimination (see Table D II 1.1) Buchan fish were also an important component of the western immatures.

These features could indicate that the big immatures in the western area belong mainly to the Buchan stock. In that case, we may conclude that partial recruitment is more pronounced in the Buchan stock than in the Bank and Downs stocks. However, it cannot be excluded that the big immatures are in fact recovering spents. Maturity determinations done on the same material by different persons have shown that it is sometimes difficult to distinguish between maturity VIII and virgin fish in maturity II.

Table C V 1 Percentage maturity composition of the herring from the five areas

Survey	Maturity stage	Area				
		Northern North Sea 1	Skagerak 2	Central West 3	Central East 4	South 5
1958 year-class						
Autumn 1960	I-II	98.7	100.0	94.2	99.4	95.8
	III-IV	1.8	-	5.8	0.5	-
	V+	-	-	-	0.05	4.2
Nos. of fish		113	1	411	1 771	24
Spring 1961	I-II	100.0	97.5	99.5	98.8	96.7
	III-IV	-	2.5	-	0.6	0.5
	V+	-	-	0.5	0.6	2.9
Nos. of fish		141	282	2 498	1 324	3 431
Autumn 1961	I-II	100.0	75.0	12.0	1.4	-
	III-IV	-	-	57.1	49.3	-
	V+	-	25.0	30.8	49.3	-
Nos. of fish		1	4	1 437	884	0
1957 year-class						
Autumn 1960	I-II	15.9	-	39.3	20.6	18.5
	III-IV	6.2	75.0	36.0	44.1	59.2
	V+	77.9	25.0	24.8	35.3	22.2
Nos. of fish		113	8	1 334	68	54

Table C V 2 Percentage maturity composition per centimetre length group (central North Sea)

Length	Central West: Nos. of fish	Central West: Maturity stage I-II	III-IV	V+	Central East: Nos. of fish	Central East: Maturity stage I-II	III-IV	V+
	1958 year-class, autumn 1961 survey							
20					2	50.0		50.0
21	3		100.0		26	3.8	65.4	30.8
22	57	12.3	73.7	14.0	118	2.5	69.5	28.0
23	270	4.4	76.3	19.3	261	2.7	56.7	40.6
24	447	8.3	62.6	29.1	236		44.5	55.5
25	367	16.9	49.9	33.2	150		36.7	63.3
26	219	19.6	34.7	45.7	72		33.3	66.7
27	19	15.6	42.2	42.2	19		26.3	73.7
28	8	25.0	50.0	25.0				
29	2			100.0				
Nos.	1 437	173	821	443	884	12	436	436
%	99.9	12.0	57.1	30.8	100.0	1.4	49.3	49.3
	1957 year-class, autumn 1960 survey							
19	2	100.0			1	100.0		
20	13	100.0			3	100.0		
21	49	91.8	8.2		4	100.0		
22	69	58.0	37.7	4.3	9	44.4	55.6	
23	243	25.5	66.7	7.8	7		85.7	14.3
24	404	34.2	47.5	18.3	15		80.0	20.0
25	338	42.6	22.2	35.2	20	10.0	30.0	60.0
26	166	36.7	10.2	53.0	6			100.0
27	44	38.6	9.1	52.3	3		33.3	66.7
28	5	20.0		80.0				
29	1			100.0				
Nos.	1 334	523	480	331	68	14	30	24
%	100.0	39.2	36.0	24.8	100.0	20.6	44.1	35.3

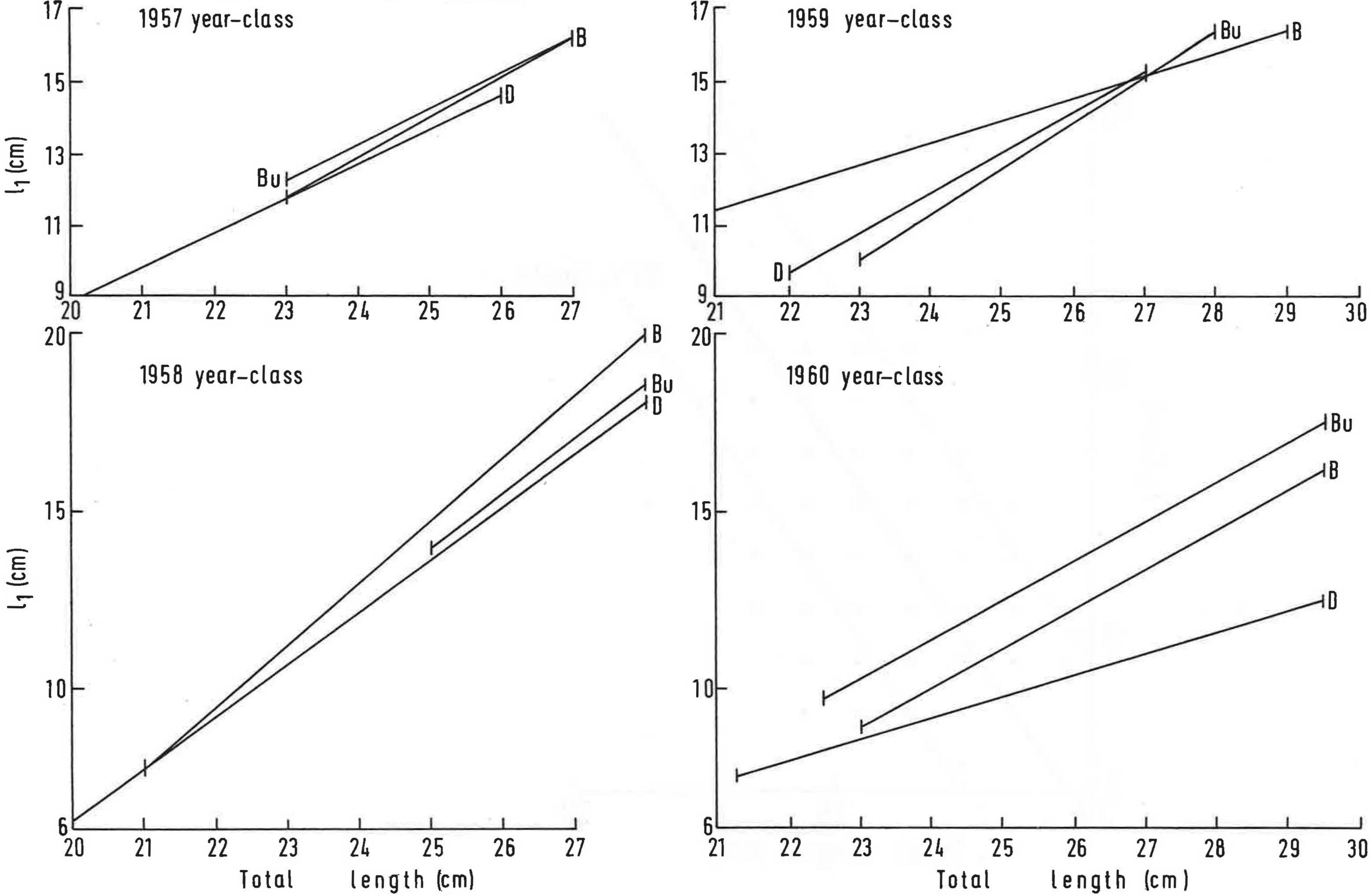


Figure C I 2 l_1/total length regressions for the Pure Stocks of the 1957, 1958, 1959 and 1960 year-classes. The regressions are only shown over the range of total lengths for which data were submitted.

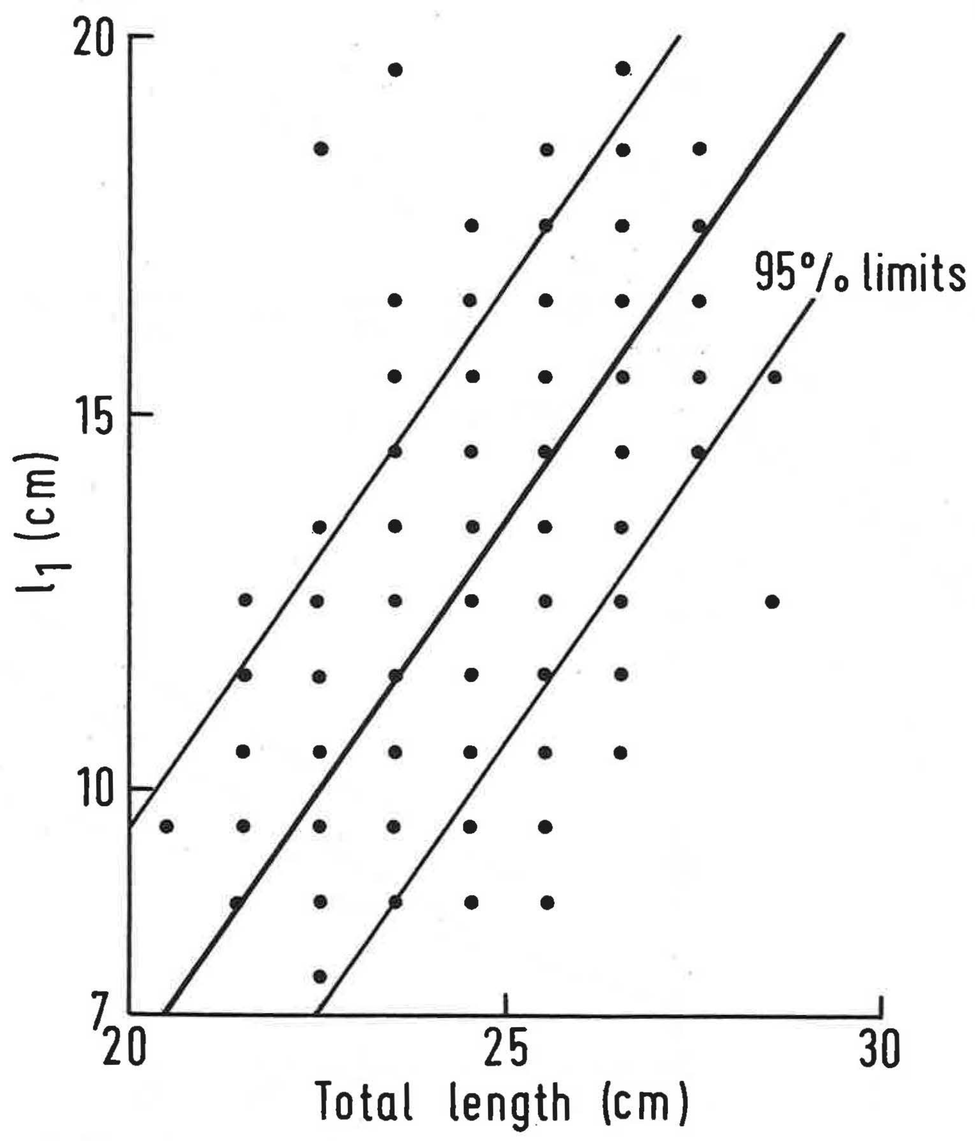


Figure C I 1 l_1/total length regression for the 1958 year-class of Downs spawning herring.

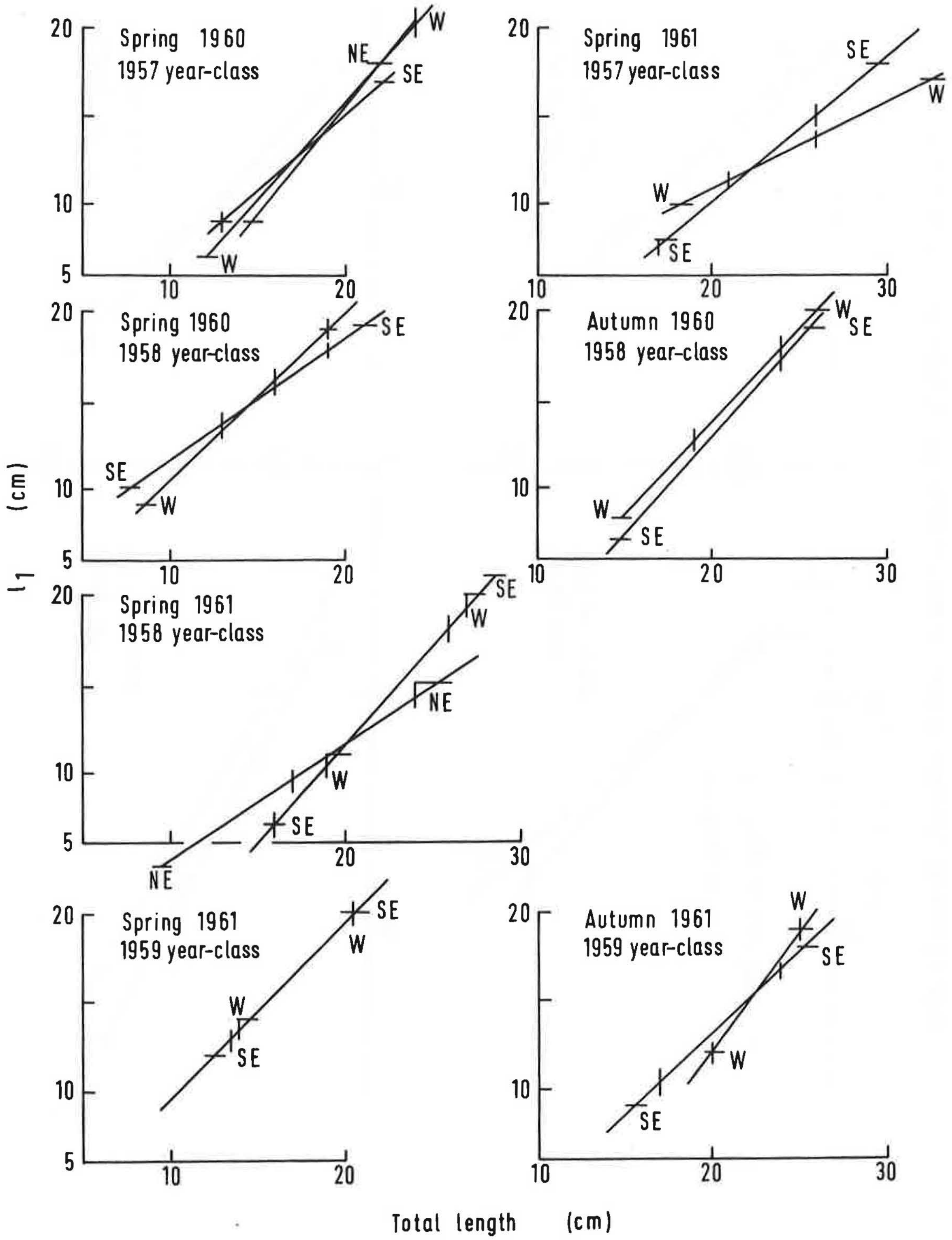


Figure C II 2 l_1/total length regressions for different areas of the North Sea.

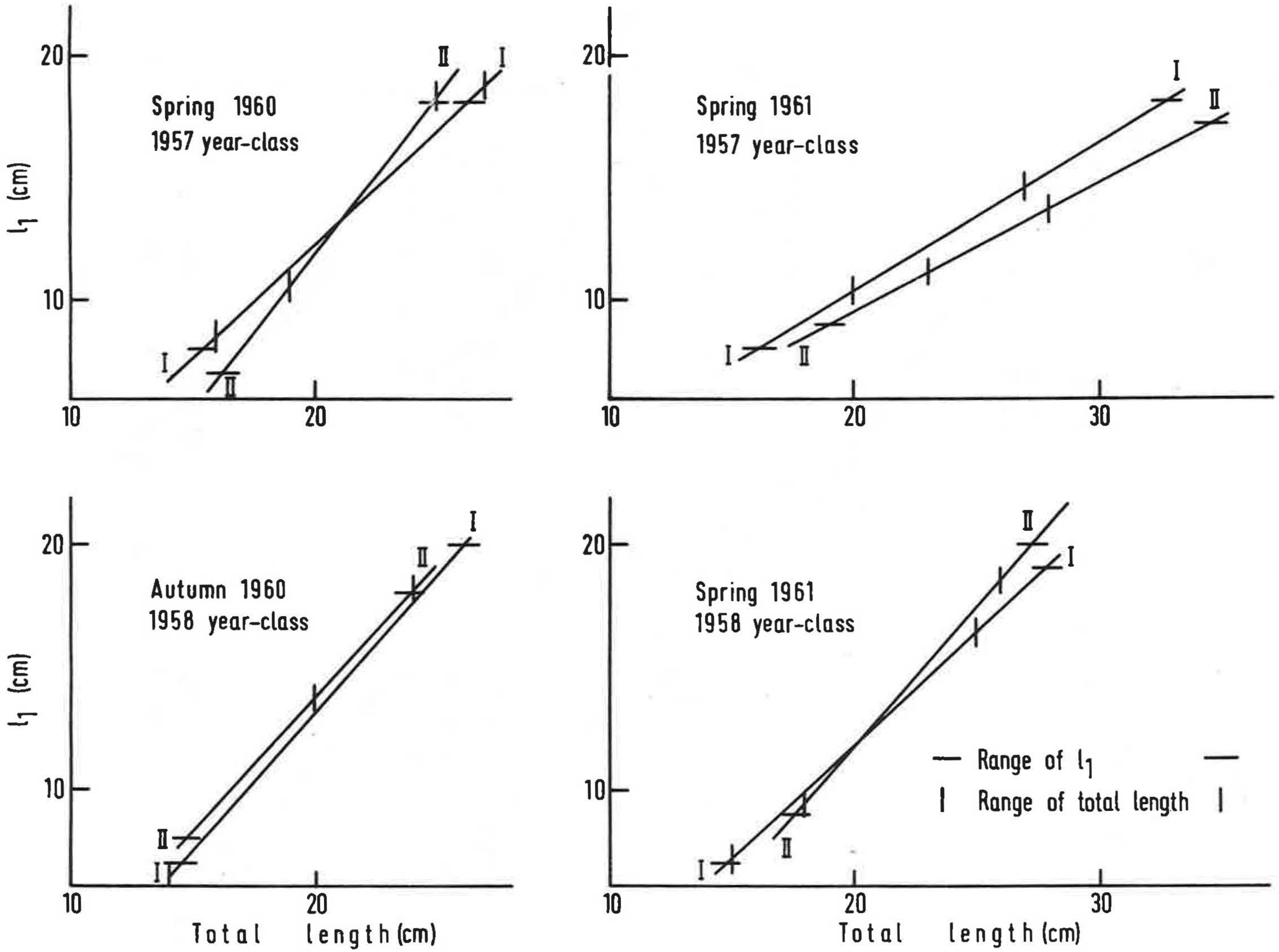


Figure C II 1 l_1/total length regressions for fish of maturity stages I and II, for the 1957 and 1958 year-classes.

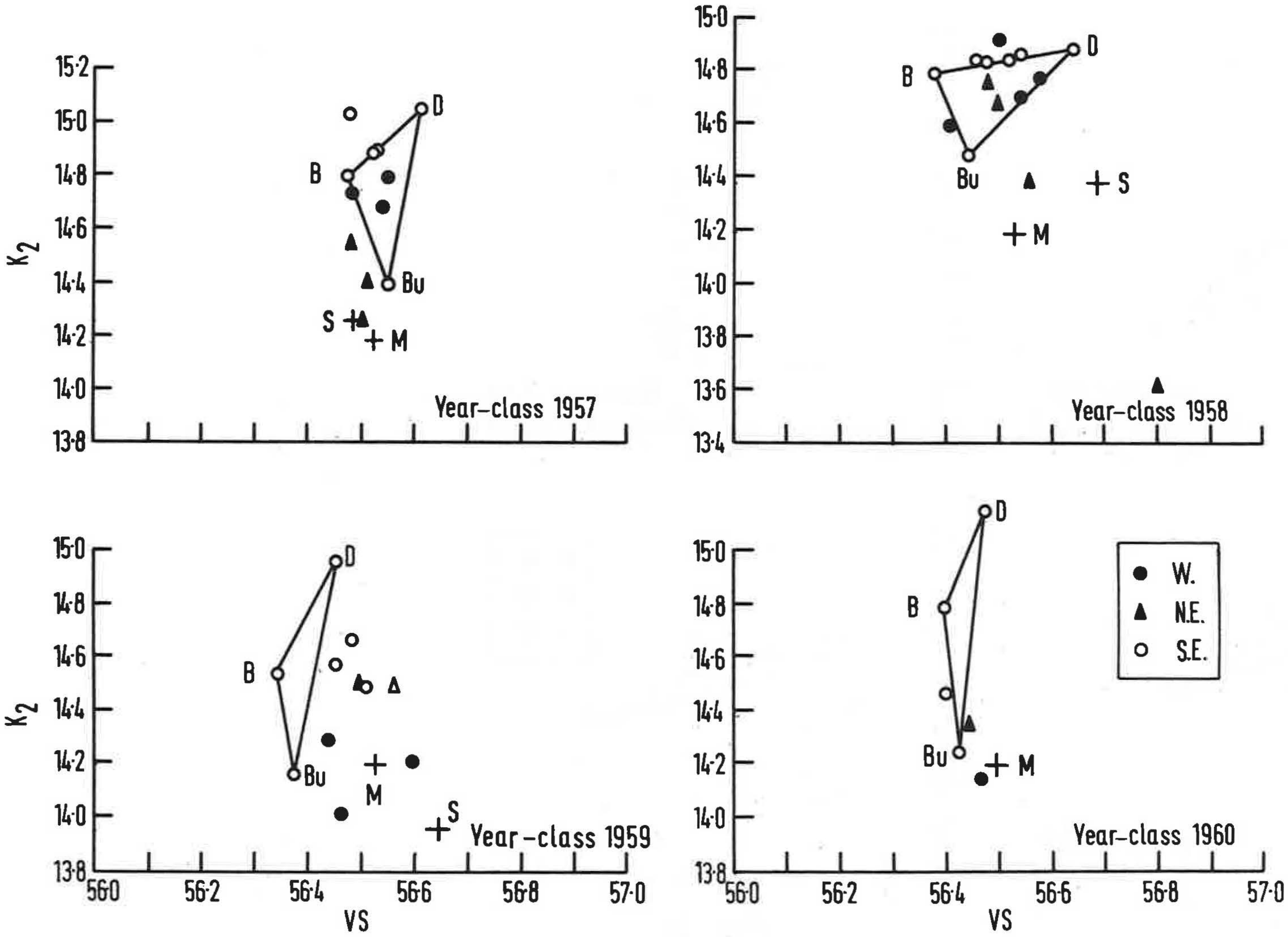


Figure C III 1 K_2/VS diagram for the Pure Stocks and the young herring of the 1957, 1958, 1959 and 1960 year-classes, and Minch (M) and Shetland (S) spawners

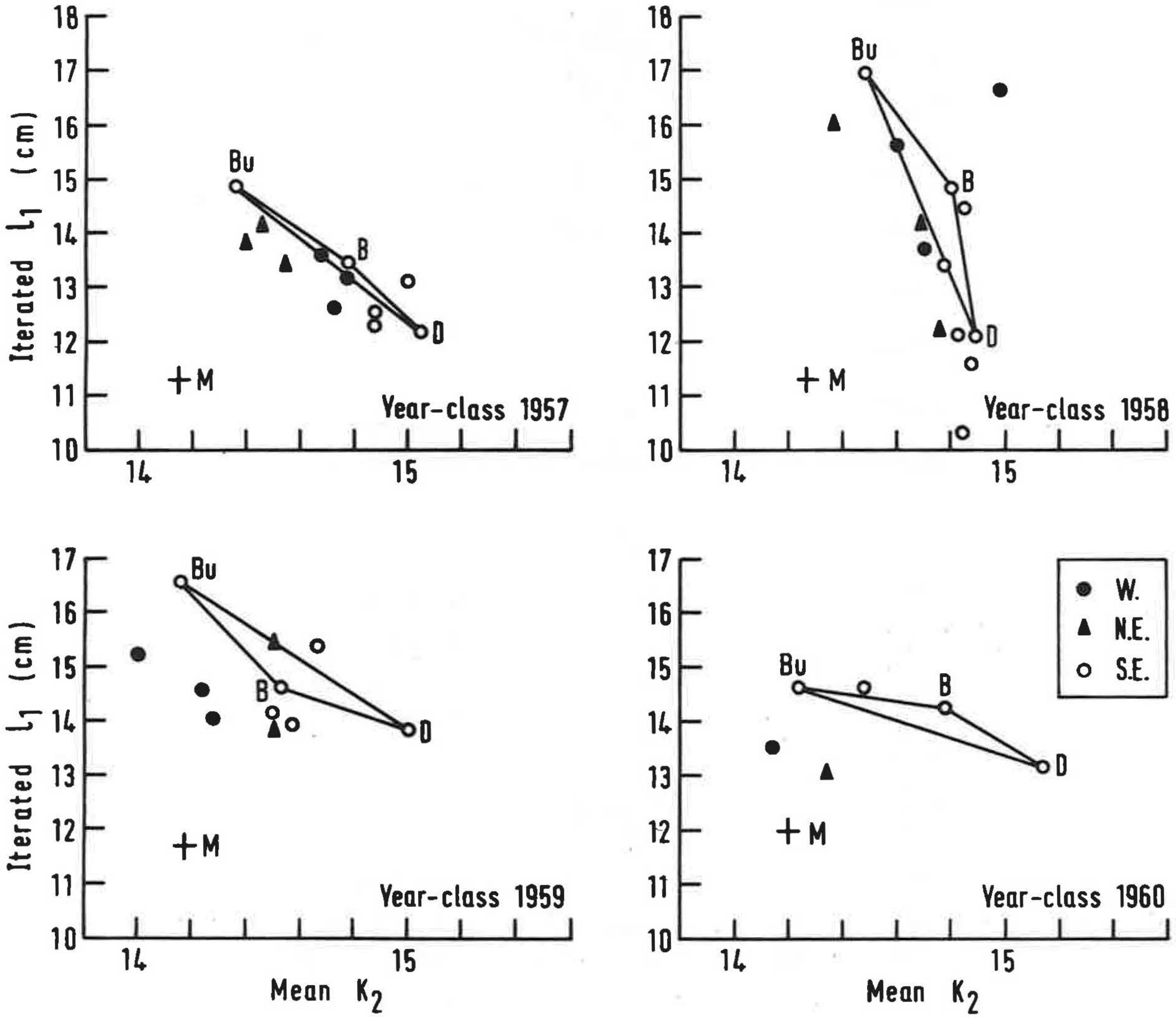


Figure C III 2 I_1/K_2 diagram for the Pure Stocks and the young herring of the 1957, 1958, 1959 and 1960 year-classes.

D RECRUITMENT PATTERN

D I Discriminant function analysis

Meristic characters have been used for many years to separate stocks of herring. However, the meristic characters show a considerable overlap between stocks and so the individual fish cannot be sorted on any one character. It would seem, however, that by using a combination of several meristic characters a better distinction between stocks could be obtained.

Discriminant function analysis is a technique devised for just such a situation, allowing a number of characters to be combined to give a linear function which can be used as a single character. The technique demands certain conditions of the characters used: (a) that each character is normally distributed, (b) that they are stochastically independent within the three stocks, and (c) that the differences between the stocks in respect of each character are due to differences in the means but that the variances are identical.

In the present case there were assumed to be three stocks in the mixed population of North Sea autumn spawners (Bank, Buchan and Downs) and three meristic characters were available to distinguish them (l_1, VS and K_2). The power of the discriminant analysis to distinguish between the stocks could have been increased by using a greater number of characters, even although each had independently little discriminant power. In practice, when the surveys were carried out it was not envisaged that this technique would be applied to the material. Thus, when the analysis came to be done only these three characters had been collected from a sufficient part of the material to be utilizable.

In the present case, with three stocks and three characters we have three discriminant functions:

$$(B/D) \quad (1) \quad X = \frac{\bar{l}_{1\,B} - \bar{l}_{1\,D}}{\text{Var}\ l_1}\, l_1 + \frac{\bar{K}_{2\,B} - \bar{K}_{2\,D}}{\text{Var}\ K_2}\, K_2 + \frac{\overline{VS}_B - \overline{VS}_D}{\text{Var}\ VS}\, VS\,,$$

$$(Bu/D) \quad (2) \quad Y = \frac{\bar{l}_{1\,Bu} - \bar{l}_{1\,D}}{\text{Var}\ l_1}\, l_1 + \frac{\bar{K}_{2\,Bu} - \bar{K}_{2\,D}}{\text{Var}\ K_2}\, K_2 + \frac{\overline{VS}_{Bu} - \overline{VS}_D}{\text{Var}\ VS}\, VS\,,$$

$$(Bu/B) \quad (3) \quad Z = \frac{\bar{l}_{1\,Bu} - \bar{l}_{1\,B}}{\text{Var}\ l_1}\, l_1 + \frac{\bar{K}_{2\,Bu} - \bar{K}_{2\,B}}{\text{Var}\ K_2}\, K_2 + \frac{\overline{VS}_{Bu} - \overline{VS}_B}{\text{Var}\ VS}\, VS\,;$$

moreover, $Z = X+Y$. This can be depicted by three splitting lines dividing a plane into three areas, each representing a stock. On the first classification, a fish is simply allocated to a stock in accordance with its X and Y. A number of fish are, of course, wrongly classified on this basis, but the probabilities of classifying each individual to each of the three stocks can be calculated from the means and variances of X and Y and their correlation coefficient.

The expectations of the first estimates of respective numbers in each stock are given by the following equations:

$$\bar{N}_D = N_D \; P_{D,D} + N_B \; P_{B,D} + N_{Bu} \; P_{Bu,D}$$

$$\bar{N}_B = N_D \; P_{D,B} + N_B \; P_{B,B} + N_{Bu} \; P_{Bu,B}$$

$$\bar{N}_{Bu} = N_D \; P_{D,Bu} + N_B \; P_{B,Bu} + N_{Bu} \; P_{Bu,Bu} ,$$

where N_D, N_B and N_{Bu} are the true abundances of the Downs, Bank and Buchan stocks and $P_{D,D}$ is the probability of classifying Downs as Downs etc. Only two of the above equations are independent, but we know that $N_D + N_B + N_{Bu} = 1$. We thus have three unknowns and three independent equations. A matrix method of obtaining nine unbiased estimates of the true population numbers from these is given in Appendix 4.

D II 1 Results of the analysis

The results of the discriminant analyses reported here and in the following section are those obtained using the iterated l_1 values given in Table C II 8, together with the VS and K_2 parameters shown in Table C I 3. It is believed that, within the limitations of the data available and the procedure chosen, the values of the Pure Stock parameters are the best that can be derived.

Discriminant analyses were made on the young herring, using the following combination of characters: l_1/VS/K_2, l_1/VS, VS/K_2 and l_1/K_2. Not all possible analyses were made for each year-class, and some gave nonsense results. The allocations were made both on the combined data from the total North Sea and also on the data grouped into the three areas, West, North-east and South-east, as shown in Figure B I 13. The detailed analyses of the data are shown in Tables D II 1.1-4. In Table D II 1.5 the covariances, as calculated in Appendix 4, are presented for all year-classes. Only the allocations for the separate areas are considered here. The total North Sea estimates given are the sums of the area estimates. In this way the maximum amount of information was utilized concerning the area distribution of the allocated stocks.

1957 year-class

The equations of the discriminant functions used in the analysis are given below:

Bank/Downs	$X = 0.7683\, l_1 - 0.3333\, VS - 0.3571\, K_2$
Downs/Buchan	$Y = -1.7124\, l_1 + 0.1429\, VS + 0.9429\, K_2$
Bank/Buchan	$Z = X + Y.$

This year-class was present in the 1960 spring and autumn surveys and in spring 1961 when it was recruiting to the adult spawning stocks. Discriminations were made using both l_1/VS/K_2 and VS/K_2 and the results are presented in Table D II 1.1. The numbers of fish as classified to the three stocks are shown for each area, together with the standard deviations of the estimates. Comparison of these two sets of estimates shows that the variances on the allocations are far greater using VS/K_2 than l_1/VS/K_2.

Table D II 1.1 Numbers and standard deviations of fish classified by discriminant function analysis; 1957 year-class

Survey		l_1/VS/K_2				VS/K_2			
		Total	D	B	Bu	Total	D	B	Bu
Spring 1960	W	399	191 29.05	48 46.00	160 23.60	399	175 97.05	68 156.00	156 70.80
	NE	500	174 31.32	95 51.50	231 27.62	500	35 105.00	130 173.50	335 82.00
	SE	485	432 34.08	neg.	53 20.72	485	131 116.93	354 155.03	neg.
	Total	1 384	797 32.22	143 50.70	444 25.85	1 384	341 105.92	552 159.82	491 78.67
Autumn 1960	W	828	546 49.64	282 52.80	neg.	828	464 116.00	neg.	364 86.63
	NE	12	neg.	3 3.59	9 4.96	12	neg.	5 2.40	7 14.08
	SE	259	220 26.92	39 27.86	neg.	259	166 79.89	28 123.96	65 56.16
	Total	1 099	766 44.62	324 50.39	9 4.96	1 099	630 107.70	33 114.22	436 82.40
Spring 1961	W	196	neg.	196 17.21	neg.	196	35 66.69	67 109.76	94 50.33
	NE	98	neg.	86 15.70	12 14.90	98	15 39.09	neg.	83 29.38
	SE	81	neg.	81 16.33	neg.	81	69 30.26	0 -	12 20.28
	Total	375	0 -	363 52.32	12 14.90	375	119 45.10	67 109.76	189 40.47

Table D II 1.2 Numbers and standard deviations of fish classified by discriminant function analysis; 1958 year-class

Survey		l_1/VS/K_2				VS/K_2			
		Total	D	B	Bu	Total	D	B	Bu
Spring 1960	W	195	neg.	39	156	195	62	133	neg.
				11.88	9.94		47.08	60.73	
	NE	826	50	99	677	826	neg.	neg.	826
			10.83	23.92	19.81				59.28
	SE	1 662	282	1 014	366	1 662	532	864	266
			26.54	41.55	28.28		134.66	207.69	136.32
	Total	2 683	332	1 152	1 199	2 683	594	997	1 092
			25.25	40.01	21.87		128.65	194.91	84.49
Autumn 1960	W	335	neg.	161	174	335	neg.	20	315
				17.39	14.05			27.40	68.36
	NE	3	neg.	3	neg.	3	neg.	3	neg.
		-		-		-		-	
	SE	1 681	1 614	neg.	67	1 681	639	992	50
			24.70		8.37		136.20	210.16	136.66
	Total	2 019	1 614	164	241	2 019	639	1 015	365
			24.70	17.39	12.62		136.20	207.66	81.01
Spring 1961	W	449	180	229	40	449	49	148	252
			16.20	21.10	11.55		66.37	103.60	77.40
	NE	318	241	51	26	318	neg.	166	152
			13.00	15.62	7.47			86.55	65.86
	SE	452	321	131	0	452	113	307	32
			16.28	18.59			70.96	110.61	74.97
	Total	1 219	742	411	66	1 219	162	621	436
			15.09	19.81	10.33		69.53	103.40	73.20
Autumn 1961	W	1 081	389	541	151	1 081	487	313	281
			24.85	32.46	18.33		108.22	165.13	112.39
	NE	5	4	neg.	1	5	neg.	neg.	5
		-	-		-	-			-
	SE	930	567	307	56	930	65	865	neg.
			23.24	288.39	14.00		100.29	156.23	
	Total	2 016	960	848	208	2 016	552	1 178	286
			23.78	175.15	17.01		107.10	158.43	112.39

In the reclassification of the first estimates some negative proportions of the components were obtained. These are indicated in the table. All these values may be taken as zeros except in spring 1961 for the $l_1/VS/K_2$ allocation. All the Downs estimates and the Buchan values for the west and south-east areas give significant negative values. It is evident that something has gone wrong. This problem does not arise in the VS/K_2 separation and it is suggested that the total lengths of the fish are the contributors to the misclassification. As has been pointed out in section C V, it is thought that partial recruitment was of considerable magnitude in this year-class, which could result in length segregation in the spring of 1961.

1958 year-class

The relevant allocations by $l_1/VS/K_2$ and VS/K_2 to the stocks are shown in Table D II 1. 2. They were obtained using the following discriminant functions:

Bank/Downs $X = 1.4204\, l_1 - 0.5909\, VS - 0.1324\, K_2$

Downs/Buchan $Y = -2.8824\, l_1 + 0.4545\, VS + 0.5735\, K_2$

Bank/Buchan $Z = X + Y.$

Inspection of the table again demonstrates great differences in the magnitudes of the variances of the two methods of allocation. In no cases are the negative values significantly different from zero. Both in the autumn of 1960 and 1961 very few fish were analysed in the north-east area (3 and 5 respectively).

1959 year-class

Discrimination of the young herring was originally attempted using $l_1/VS/K_2$ and VS/K_2 as had been done for the two previous year-classes. No estimates could be obtained using these functions in the 1959 year-class. The functions used were:

Bank/Downs $X = 0.4363\, l_1 - 0.2200\, VS - 0.5942\, K_2$

Downs/Buchan $Y = -0.7794\, l_1 + 0.1600\, VS + 1.1449\, K_2$

Bank/Buchan $Z = X + Y.$

In section C III it was seen from the VS/K_2 plot, Figure C III 1, that the mean values of the young herring were poorly associated with the loci of the Pure Stock estimates. As has been mentioned, it was thought unlikely that the determinations of VS and K_2 were wrong but that the problem lay in the underlying assumptions concerning the Pure Stocks. Inspection of Figure C III 1 suggested that far greater numbers of high vertebral count fish were present among the young herring than could be accounted for from the values of the Pure Stocks available. However, it was also clear that in the case of K_2 the values of the young herring were not inconsistent with those of the Pure Stocks. A discrimination using l_1/K_2 was made, the results of which appear in Table D II 1. 3.

Though the analysis of this year-class is far from satisfactory, some allocations for the different areas have been derived. However, Figure C III 2 shows that the l_1 and K_2 values of the young herring could be associated with an l_1/K_2 diagram which includes the values for Minch spawning herring. The contrast between the breakdown of the discriminant analyses using $l_1/VS/K_2$ and

Table D II 1.3 Numbers and standard deviations of fish classified by discriminant function analysis; 1959 year-class

Survey		l_1/K_2 Total	D	B	Bu
Autumn 1960	W	143	0	0	143 47.20
	NE	0	neg.	neg.	
	SE	216	108 52.94	108 19.71	
	Total	359	108 52.94	108 19.71	143 47.20
Spring 1961	W	322	0	neg.	322 39.03
	NE	463	0	neg.	463 104.73
	SE	424	204 42.34	neg.	220 68.95
	Total	1 209	204 42.34	0	1 005 81.30
Autumn 1961	W	168	neg.	84 65.19	84 75.16
	NE	0	0	0	0
	SE	188	188 25.49	neg.	0
	Total	356	188 25.49	84 65.19	84 75.16

VS/K_2 and the statistically acceptable analysis by l_1/K_2 initiated the enquiry into the possibility of the existence of a fourth major component among the young herring.

Though a statistically acceptable discrimination had been made, the presence of this fourth component would render the analysis unreliable for making proper assessments of the true abundance of Bank and Buchan stocks in this year-class; it is likely that the estimate for the Downs component is little affected by the misassignments among the low K_2, high VS and high l_1 fish in the juvenile herring.

1960 year-class

It can be seen from Figure C III 1 that VS provides little discrimination between the stocks in this year-class. Both this figure and Figure C III 2 show that the greatest discriminant power lies in the K_2 values. This is also seen in the equations of the functions:

Bank/Downs $X = 0.6327\, l_1 - 0.1770\, VS - 0.5450\, K_2$

Downs/Buchan $Y = -0.8329\, l_1 + 0.1106\, VS + 1.3624\, K_2$

Bank/Buchan $Z = X + Y.$

For this reason VS was discarded, and the allocations presented in Table D II 1.4 are based on an l_1/K_2 analysis. This year-class occurred only as 0-group fish in the 1961 autumn survey. It is unlikely that each of the three stocks is fully represented. None of the negative values is significantly different from zero.

Table D II 1.4 Numbers and standard deviations of fish classified by discriminant function analysis; 1960 year-class

Survey		l_1/K_2			
		Total	D	B	Bu
Autumn 1961	W	48	12 4.00	neg.	36 4.23
	NE	215	114 16.06	neg.	101 28.86
	SE	1 757	neg.	949 135.57	808 104.73
	Total	2 020	126 15.27	949 135.57	945 97.55

Table D II 1.5 Estimates of covariances ($\hat{n}_D$ $\hat{n}_B$)

Year-class	Survey	$l_1/VS/K_2$			VS/K_2			l_1/K_2		
		W	NE	SE	W	NE	SE	W	NE	SE
1957	Spring 1960	-0.0076	-0.0058	-0.0070	-0.0903	-0.0686	-0.0881			
	Autumn 1960	-0.0052	-0.1957	-0.0153	-0.0407	-3.1114	-0.1426			
	Spring 1961	-0.0158	-0.0301	-0.0563	-0.1805	-0.3048	-0.4153			
1958	Spring 1960	-0.00094	-0.00023	-0.00030	-0.0700	-0.0094	-0.0078			
	Autumn 1960	-0.00085	-	-0.00026	-0.02988	-2.4262	0.00776			
	Spring 1961	-0.00142	-0.00178	-0.00151	-0.02521	-0.03644	-0.02872			
	Autumn 1961	-0.00056	-	-0.00070	-0.01134	-	-0.01444			
1959	Autumn 1960							-0.1091	-	-0.1064
	Spring 1961							-0.0395	-0.0355	-0.0388
	Autumn 1961							-0.1129	-	-0.0885
1960	Autumn 1961							Total North Sea -0.0030		

D II 2 Interpretation of the classifications obtained

Except for the 1957 year-class in the spring of 1961, using the $l_1/VS/K_2$ classification the numbers of fish given in Tables D II 1.1-4 can be taken as estimates of the proportions, $\hat{n}$, of the three stocks in the young herring. We can now examine the agreement between the results of the $l_1/VS/K_2$ and VS/K_2 analyses.

As previously mentioned, the two sets of $\hat{n}$'s are approximately independent, one being based mainly on l_1 and the other on VS/K_2. If the two separations are identical the two-dimensional variable

$$(\Delta\hat{n}_D, \Delta\hat{n}_B) = \left[\hat{n}_{D(l_1/VS/K_2)} - \hat{n}_{D(VS/K_2)}, \hat{n}_{B(l_1/VS/K_2)} - \hat{n}_{B(VS/K_2)}\right] \tag{1}$$

is approximately normally distributed about (0, 0) with variances and covariances as below:

$$\mathrm{Var}\,\Delta\hat{n} \approx \mathrm{Var}\,\hat{n}_{(l_1/VS/K_2)} + \mathrm{Var}\,\hat{n}_{(VS/K_2)} \tag{2}$$

$$\mathrm{Cov}\,(\Delta\hat{n}_D, \Delta\hat{n}_B) = \mathrm{Cov}\left[\hat{n}_{D(l_1/VS/K_2)}, \hat{n}_{B(l_1/VS/K_2)}\right] + \mathrm{Cov}\left[\hat{n}_{D(VS/K_2)}, \hat{n}_{B(VS/K_2)}\right]. \tag{3}$$

This means that

$$\frac{1}{\mathrm{Var}\,\Delta\hat{n}_D \cdot \mathrm{Var}\,\Delta\hat{n}_B - \mathrm{Cov}^2(\Delta\hat{n}_D, \Delta\hat{n}_B)}\left[\mathrm{Var}\,\Delta\hat{n}_B \cdot (\Delta\hat{n}_D)^2 - 2\,\mathrm{Cov}\,(\Delta\hat{n}_D, \Delta\hat{n}_B) \cdot \Delta\hat{n}_D \cdot \Delta\hat{n}_B + \mathrm{Var}\,\Delta\hat{n}_D \cdot (\Delta\hat{n}_B)^2\right]$$

is approximately χ^2 distributed with 2 degrees of freedom under the hypothesis that

$$E\left[\hat{n}_{(l_1/VS/K_2)}\right] = E\left[\hat{n}_{(VS/K_2)}\right] = \text{true n.}$$

The following approximate χ^2 values are found by using estimates of variances and covariances:

	1957 year-class			1958 year-class		
	W	NE	SE	W	NE	SE
Spring 1960	0.03	10.4	6.7	17.7	59.1	4.4
Autumn 1960	50.1		7.5	5.1		53.7
Spring 1961	8.8	5.3	6.9	12.4	26.6	9.1
Autumn 1961				2.1		23.8

As $\chi^2_{95\%} = 5.99$, the table shows clear discrepancies between the two sets of estimates.

Assuming the VS/K_2 proportions to be true it is possible to calculate what the expected discrimination would be when using the $l_1/VS/K_2$ function. These estimates, n^*, are derived from Table D II 2.1, together with the estimates of $\hat{n}$ from VS/K_2 and n_2^* from $l_1/VS/K_2$.

As the VS/K_2 characters are meristic characters free from the effects of length bias, one might suppose that these separations are unbiased even though they do have a great variance. Because of this variance they are not well suited for estimating the proportions of the stocks. However, from inspection of Table D II 2.1 when comparing n_1^* with n_2^* it would appear that n_1^* gives a greater proportion of Buchan fish than those found by the $l_1/VS/K_2$ analysis. The Downs proportion is correspondingly lower. It would seem that the introduction of l_1 in the $l_1/VS/K_2$ discrimination has shifted the splitting lines towards the Buchan stock centre (Appendix 4, Figure 2), thus overestimating the proportion of Downs fish.

It would further appear that this shift has occurred in the same direction in all three sub-areas of the North Sea, equally as much in those areas of large fish, such as the west, as in the south-east area where smaller fish occur. This then does not indicate any clear segregation by length.

The position, therefore, is that the $l_1/VS/K_2$ discrimination is biased towards the Downs stock, but the VS/K_2 discrimination has such a high variance that the significance of the stock allocations is low. In the absence of any simple way of resolving this problem, we will present stock abundance indices derived from the two separations and raised by three sets of abundance indices.

Table D II 2.1 Estimates of the stock allocations for the 1957 and 1958 year-classes in each area and survey. For spring 1960 $\hat{n}$ is derived from Table D III 2 and n_2^* from Table D III 1

$\hat{n}$ from VS/K_2
n_1^* calculated $l_1/VS/K_2$ separations (see text)
n_2^* from $l_1/VS/K_2$

Survey	Area		D	B	Bu
	1957 year-class				
Spring 1960	W	$\hat{n}$	0.44	0.17	0.39
		n_1^*	0.37	0.27	0.36
		n_2^*	0.39	0.26	0.35
	NE	$\hat{n}$	0.07	0.26	0.67
		n_1^*	0.16	0.28	0.56
		n_2^*	0.32	0.27	0.41
	SE	$\hat{n}$	0.27	0.73	-
		n_1^*	0.41	0.38	0.21
		n_2^*	0.63	0.24	0.13
Autumn 1960	W	$\hat{n}$	0.56	-	0.44
		n_1^*	0.41	0.24	0.35
		n_2^*	0.60	0.34	0.06
	SE	$\hat{n}$	0.64	0.11	0.25
		n_1^*	0.49	0.26	0.25
		n_2^*	0.65	0.29	0.06
Spring 1961	W	$\hat{n}$	0.18	0.34	0.48
		n_1^*	0.25	0.30	0.45
		n_2^*	0.10	0.62	0.28
	NE	$\hat{n}$	0.15	-	0.85
		n_1^*	0.14	0.22	0.64
		n_2^*	0.06	0.50	0.44
	SE	$\hat{n}$	0.85	-	0.15
		n_1^*	0.60	0.25	0.15
		n_2^*	0.36	0.59	0.05

Table D II 2.1 continued

Survey	Area		D	B	Bu
	1958 year-class				
Spring 1960	W	$\hat{n}$	0.32	0.68	-
		n^*_1	0.38	0.50	0.12
		n^*_2	0.03	0.27	0.70
	NE	$\hat{n}$	-	-	1.00
		n^*_1	-	0.17	0.83
		n^*_2	0.07	0.23	0.70
	SE	$\hat{n}$	0.32	0.52	0.16
		n^*_1	0.35	0.43	0.22
		n^*_2	0.25	0.47	0.28
Autumn 1960	W	$\hat{n}$	-	0.06	0.94
		n^*_1	0.01	0.19	0.80
		n^*_2	0.08	0.41	0.51
	SE	$\hat{n}$	0.38	0.59	0.03
		n^*_1	0.42	0.46	0.12
		n^*_2	0.81	0.16	0.03
Spring 1961	W	$\hat{n}$	0.11	0.33	0.56
		n^*_1	0.15	0.33	0.52
		n^*_2	0.42	0.42	0.16
	NE	$\hat{n}$	-	0.52	0.48
		n^*_1	0.09	0.42	0.49
		n^*_2	0.66	0.25	0.09
	SE	$\hat{n}$	0.25	0.68	0.07
		n^*_1	0.32	0.51	0.17
		n^*_2	0.64	0.32	0.04
Autumn 1961	W	$\hat{n}$	0.45	0.29	0.26
		n^*_1	0.42	0.31	0.27
		n^*_2	0.39	0.41	0.20
	SE	$\hat{n}$	0.07	0.93	-
		n^*_1	0.22	0.63	0.15
		n^*_2	0.57	0.33	0.10

D III Estimates of stock abundances

The stock assignments in Tables D II 1.1-4 refer to the fish sampled biologically. It is possible to regard these as random samples of the young herring population in the North Sea. However, it is also possible to use the abundance indices by areas to weight the area discriminations to give other estimates of the stock sizes in the North Sea. The most comprehensive estimates of abundance are those presented in section B IV. These were derived from a different area separation of the North Sea from that used in the discriminant analysis. It is necessary, therefore, to derive from these five areas abundance indices applicable to the three standard areas. These abundance indices have been calculated on the basis of the ratio of the numbers of 30 nautical mile rectangles of each of the five areas which occur in the three standard areas. The following equations give the relationships between these standard areas and the five areas:

$$\text{West} = 0.5410\ a_1 + 0.7778\ a_3 + 0.3333\ a_5$$

$$\text{North-east} = 0.4590\ a_1 + a_2$$

$$\text{South-east} = 0.2222\ a_3 + a_4 + 0.6667\ a_5\ ,$$

where a_1, a_2, a_3, etc. are the abundances of a year-class in the areas of the same designation in Figure B I 13.

In applying the abundance indices to the stock discriminations some redistribution has been made of the stock components. As has been pointed out previously in the year-class analyses, where negative values have been found which do not differ significantly from zero, those stock components have been regarded as absent and the proportion has been redistributed between the other two components.

Table D III 1 gives these adjusted abundance indices of the 1957 year-class for the two discriminations $l_1/VS/K_2$ and VS/K_2. The abundance indices in this table include abundance adjustment from the Bløden fishery (see Appendix 3). Table D III 2 gives the estimates of stocks when all ships within the surveys have been assumed to have the same power factor. These abundance indices are comparable both between stocks within a survey and between surveys. In both tables estimates of the mean standard deviation for the stock separations shown in Table D II 1 are expressed as percentages. Additionally, the standard deviations for the abundance indices for each survey are shown. These are derived in Appendix 3, Tables 4 and 8.

The large standard deviation of the stock allocation by VS/K_2 has been commented upon previously and contrasts sharply with that using $l_1/VS/K_2$. The abundance indices have standard deviations of between 13 and 48 per cent of the means. The stock separations can thus never be very precise ones.

In contrast the abundance indices derived in section B I 1 have also been used. These do not include the index from the Bløden fishery. They give a comparison between stocks within a survey. If it is assumed that the fishing power of the vessels does not change with time it is possible to compare the stock abundances between surveys. These abundance indices are given in Table D III 3.

However, unlike the data of Tables D III 1 and 2, no confidence limits can be placed on these estimates, since the variance is indeterminate from the method used to derive the indices.

The comparable abundance indices for the 1958 year-class are given in Tables D III 4, 5 and 6. Again the standard deviations available are shown with the indices. It is clear that the best estimate, giving the least standard deviation, is that derived from $l_1/VS/K_2$.

The abundance indices for the 1960 year-class appear in Table D III 7. The very high numbers of Downs stock of this year-class found in the north-eastern region in abundance indices 1 and 3 reflect the large catch made by one of the research vessels in the autumn 1961 cruise in one statistical rectangle in the vicinity of the Skagerak; 46 per cent of the fish used in the biological sample for the discrimination came from this catch. In the estimate including the research vessel power factor, the numbers of fish designated as Downs are likely to be unduly weighted by the size of this one catch. Both estimates 2 and 3 indicate that the Buchan component is the largest in this year-class and the Downs small. However, as the fish are 0-group there might have been further emigration of Downs and Bank stocks from the coastal areas in the following winter, so that the relative proportions of the stocks might not represent the differences in real stock abundance.

For the 1957 and 1958 year-classes all the estimates made may be taken as equally good estimates of the indices of stock size; the seasonal estimates have been meaned, and overall abundances obtained for each stock. In Table D III 8 are given two estimates, those derived from Tables D III 1 and 3 for the 1957 year-class, those from Tables D III 4 and 6 for the 1958 year-class. Relative year-class strengths may be calculated by taking the two year-classes at comparable ages. Thus the 1957 year-class in the spring and autumn surveys of 1960 may be compared with the 1958 year-class in the spring and autumn of 1961. This comparison is made in Table D III 9. The relative estimates of year-class strengths of the Bank, Downs and Buchan stocks are rather similar in both methods.

Table D III 1 Abundance indices for the 1957 year-class, with adjustment from the Bløden fishery, with the between-ship power factor included (Appendix 3, Table 6)

Survey	Standard deviation of abundance index (%)		Total	$1_1/VS/K_2$			VS/K_2		
				D	B	Bu	D	B	Bu
Spring 1960	± 47.6	W	27 212	13 062	3 265	10 885	11 973	4 626	10 613
		NE	327	114	62	151	23	85	219
		SE	8 823	7 852	-	971	2 382	6 441	-
		Total	36 362	21 028	3 327	12 007	14 378	11 152	10 832
Autumn 1960	± 11.6	W	1 403	926	477	-	786	-	617
		NE	187	-	47	140	-	79	108
		SE	910	774	136	-	582	100	228
		Total	2 500	1 700	660	140	1 368	179	953
Spring 1961	± 29.9	W	539	-	539	-	97	183	259
		NE	91	-	80	11	14	-	77
		SE	301	-	301	-	256	-	45
		Total	931	-	920	11	367	183	381
Standard deviation of stock allocation (%)				± 5	± 22	± 6	± 29	± 97	± 19

Table D III 2 Abundance indices for the 1957 year-class, with adjustment from the Bløden fishery, but without research vessel power factors (Appendix 3, Table 9)

Survey	Standard deviation of abundance index (%)		Total	1_1/VS/K_2			VS/K_2		
				D	B	Bu	D	B	Bu
Spring 1960	± 19.4	W	1 664	799	200	666	732	283	649
		NE	557	195	106	256	39	145	373
		SE	1 032	918	-	114	279	753	-
		Total	3 253	1 912	306	1 036	1 050	1 181	1 022
Autumn 1960	± 12.5	W	1 279	844	435	-	716	-	563
		NE	68	-	17	51	-	29	39
		SE	916	779	137	-	586	101	229
		Total	2 263	1 623	589	51	1 302	130	831
Spring 1961	± 47.8	W	474	-	474	-	85	161	228
		NE	92	-	81	11	14	-	78
		SE	266	-	266	-	226	-	40
		Total	832	-	821	11	325	161	346
Standard deviation of stock allocation (%)				± 5	± 22	± 6	± 29	± 97	± 19

Table D III 3 Abundance indices using the method of section B I 1, 1957 year-class. The standard deviations of the abundance index are indeterminate

Survey	Standard deviation of abundance index (%)		Total	$l_1/VS/K_2$			VS/K_2		
				D	B	Bu	D	B	Bu
Spring 1960	indet.	W	15 291	7 340	1 835	6 116	6 729	2 599	5 963
		NE	4 323	1 513	821	1 989	303	1 124	2 896
		SE	7 310	6 506	-	804	1 974	5 336	-
		Total	26 924	15 359	2 656	8 909	9 006	9 059	8 859
Autumn 1960	indet.	W	5 014	3 309	1 705	-	2 808	-	2 206
		NE	53	-	13	40	-	22	31
		SE	2 721	2 313	408	-	1 742	299	680
		Total	7 788	5 622	2 126	40	4 550	321	2 917
Spring 1961	indet.	W	2 022	-	2 022	-	364	687	971
		NE	228	-	201	27	34	-	194
		SE	657	-	657	-	558	-	99
		Total	2 907	-	2 880	27	956	687	1 264
Standard deviation of stock allocation (%)				± 5	± 22	± 6	± 29	± 97	± 19

Table D III 4 Abundance indices for the 1958 year-class, with adjustment from the Bløden fishery, with the between-ship power factor included (Appendix 3, Table 6)

Survey	Standard deviation of abundance index (%)		Total	$l_1/VS/K_2$			VS/K_2		
				D	B	Bu	D	B	Bu
Spring 1960	± 47.6	W	16 743	-	3 349	13 394	5 358	11 385	-
		NE	582	35	70	477	-	-	582
		SE	27 339	4 648	16 677	6 015	8 748	14 216	4 374
		Total	44 664	4 683	20 096	19 886	14 106	25 601	4 956
Autumn 1960	± 11.6	W	350	-	168	182	-	21	329
		NE	83	-	83	-	-	83	-
		SE	10 167	9 760	-	407	3 863	5 999	305
		Total	10 600	9 760	251	589	3 863	6 103	634
Spring 1961	± 29.9	W	1 739	696	887	157	191	574	974
		NE	534	406	85	43	-	278	256
		SE	2 680	1 903	777	-	670	1 822	188
		Total	4 953	3 005	1 749	200	861	2 674	1 418
Autumn 1961	± 59.7	W	3 809	1 371	1 904	533	1 714	1 105	990
		NE	204	161	-	43	-	-	204
		SE	1 852	1 130	611	111	130	1 722	-
		Total	5 865	2 662	2 515	687	1 844	2 827	1 194
Standard deviation of stock allocation (%)				± 4	± 8	± 8	± 26	± 18	± 22

Table D III 5 Abundance indices for the 1958 year-class, with adjustment from the Bløden fishery, but without research vessel power factors (Appendix 3, Table 9)

Survey	Standard deviation of abundance index (%)		Total	l_1/VS/K_2			VS/K_2		
				D	B	Bu	D	B	Bu
Spring 1960	± 19.4	W	1 379	-	276	1 103	441	938	-
		NE	1 171	70	141	960	-	-	1 171
		SE	13 806	2 347	8 422	3 037	4 417	7 179	2 209
		Total	16 356	2 417	8 839	5 100	4 858	8 117	3 380
Autumn 1960	± 12.5	W	292	-	140	152	-	18	274
		NE	29	-	29	-	-	29	-
		SE	10 170	9 763	-	407	3 865	6 000	305
		Total	10 491	9 763	169	559	3 865	6 047	579
Spring 1961	± 47.8	W	1 496	598	763	135	165	494	838
		NE	523	397	83	42	-	272	251
		SE	2 449	1 739	710	-	612	1 665	171
		Total	4 468	2 734	1 556	177	777	2 431	1 260
Autumn 1961	± 31.4	W	4 453	1 603	2 226	623	2 004	1 291	1 158
		NE	53	42	-	11	-	-	53
		SE	2 032	1 240	671	122	142	1 890	-
		Total	6 538	2 885	2 897	756	2 146	3 181	1 211
Standard deviation of stock allocation (%)				± 4	± 8	± 8	± 26	± 18	± 22

Table D III 6 Abundance indices using the method of section B I 1, 1958 year-class. The standard deviations of the abundance index are indeterminate

Survey	Standard deviation of abundance index (%)		Total	$l_1/VS/K_2$			VS/K_2		
				D	B	Bu	D	B	Bu
Spring 1960	indet.	W	4 375	-	875	3 500	1 400	2 975	-
		NE	8 950	537	1 074	7 339	-	-	8 950
		SE	151 486	25 753	92 406	33 327	48 476	78 772	24 238
		Total	164 811	26 290	94 355	44 166	49 876	81 747	33 188
Autumn 1960	indet.	W	1 717	-	824	893	-	103	1 614
		NE	25	-	25	-	-	25	-
		SE	54 765	52 574	-	2 191	20 811	32 311	1 643
		Total	56 507	52 574	849	3 084	20 811	32 439	3 257
Spring 1961	indet.	W	5 722	2 289	2 918	515	629	1 888	3 205
		NE	3 914	2 975	626	313	-	2 035	1 879
		SE	9 417	6 686	2 731	-	2 354	6 404	659
		Total	19 053	11 950	6 275	828	2 983	10 327	5 743
Autumn 1961	indet.	W	20 914	7 529	10 457	2 928	9 411	6 065	5 438
		NE	968	765	-	203	-	-	968
		SE	3 903	2 381	1 288	234	273	3 630	-
		Total	25 785	10 675	11 745	3 365	9 684	9 695	6 406
Standard deviation of stock allocation (%)				± 4	± 8	± 8	± 26	± 18	± 22

Table D III 7 Abundance indices for the 1960 year-class in autumn 1961; 1: with adjustment from the Bløden fishery with the between-ship power factor included; 2: using the abundance indices of section B I 1; and 3: with the adjustment from the Bløden fishery but without the research vessel power factors

Abundance index		Total	l_1/K_2		
			D	B	Bu
1	W	408	98	0	310
	NE	67 422	35 734	0	31 688
	SE	22 780	0	12 301	10 479
	Total	90 610	35 832	12 301	42 477
2	W	353	85	0	268
	NE	12 472	6 610	0	5 862
	SE	22 530	0	12 166	10 364
	Total	35 355	6 695	12 166	16 494
3	W	952	228	0	724
	NE	35 060	18 582	0	16 478
	SE	78 769	0	42 535	36 234
	Total	114 781	18 810	42 535	53 436

Table D III 8 — Mean year-class strengths for Bank, Downs and Buchan stocks from discriminant analysis
A from Tables D III 1 and 3
B from Tables D III 4 and 6

Year-class		$l_1/VS/K_2$			VS/K_2		
		B	D	Bu	B	D	Bu
1958	A	28 306	25 372	12 861	33 553	20 839	12 148
	B	6 163	5 028	5 330	9 301	5 169	2 051
1957	A	2 554	6 994	2 992	3 355	4 837	4 346
	B	1 636	7 575	4 052	3 838	5 370	4 055

Table D III 9 — Mean year-class strengths from discriminant analysis, taking the 1957 and 1958 year-classes at comparable ages
A from Tables D III 1 and 3
B from Tables D III 4 and 6

Year-class		$l_1/VS/K_2$			VS/K_2		
		B	D	Bu	B	D	Bu
1957	A	2 391	10 491	4 475	4 690	6 777	5 888
	B	1 994	11 364	6 073	5 666	7 873	5 898
1958	A	9 010	11 313	2 097	6 334	10 012	6 074
	B	2 834	2 154	422	1 353	2 750	1 306

D IV The young herring population in the eastern North Sea

1. Stock composition

Because of the young herring fishery in the Bløden area special attention is focused on the south-eastern area. In fact the undertaking of the surveys in 1960 and 1961 was initiated by questions raised in connection with the interpretation of the results of the ICES Bløden tagging experiment.

One of the original objectives of the young herring surveys was to obtain information on the stock composition of the young herring concentrations. This objective was not very clearly defined at the time when the surveys were carried out. In the case of the South-east area two questions may be posed:

(a) In what proportions are the stock components present in the young herring population? The discriminant function can give some estimate for the two years of survey, but it is obvious that the composition will change from year to year according to the relative year-class strengths in the three main stocks. For this reason, and because at any one time the young herring population comprises more than one year-class and the discriminant method did not succeed in one of the year-classes, no attempt is made to answer the question.

(b) How big a proportion of each stock is present in the area in different periods? In this version the question may give an answer of more general application. The stock allocation obtained by the discriminant analyses makes it possible to follow one year-class in one stock through its immature stage and to estimate its role played in the eastern central North Sea. The following attempt is based upon the discriminant analysis only using VS and K_2. As stated in section D III, this estimate is supposed to be free from the effects of length bias. Of the four year-classes involved the use of the VS/K_2 function excludes the 1959 and 1960 year-classes for the reasons given in section D III.

The 1958 year-class, being present in all four surveys, covers the I- and II-group stages, whereas the 1957 year-class only adds information to the latter. Unfortunately the 0-group stage cannot be dealt with in a comparable manner. In Table D IV 1 the percentage of the total abundance present in the South-east area is given for each stock component, using the abundance indices of Tables D III 1 and 4. The main picture brought out shows similar trends in the young herring of the Downs and Bank stocks. As I-group in spring they are found both east and west of the Dogger. In the following autumn at approximately 2 years of age both stocks seem to be concentrated exclusively in the South-east area. From here an emigration takes place, mainly towards the west, during the following winter. The last stage considered is II-group fish in autumn, which is synonymous with 3-year-old recruit spawners. According to Table C V 1 only 1.4 per cent of the 1958 year-class were still immature in the South-east area at this age. It appears that a bigger proportion of the Bank stock is present as 3-year-olds in the South-east area than is the case in the Downs and Buchan stocks. This is compatible with the fact that the spawning grounds of the Bank stock are situated very close to and partly in the South-east area. It also appears that young herring of the Buchan stock have a distinct "South-east period" but that this occurs at an earlier stage, i.e. as I-group in spring.

Table D IV 1 Percentages of the 1957 and 1958 year-classes in the South-east area (see also Figure D IV 1)

Stock	Year-class	I-group		II-group	
		Spring	Autumn	Spring	Autumn
Downs	1957	-	-	17	43
	1958	62	100	78	7
	Mean	62	100	48	25
Bank	1957	-	-	58	56
	1958	56	98	68	61
	Mean	56	98	63	59
Buchan	1957	-	-	0	24
	1958	88	48	13	0
	Mean	88	48	7	12

This difference in timing is compatible with the bigger size of the Buchan fish and its spawning being 2-5 months earlier than in the other two stocks. Considering the big variances of the VS/K_2 discrimination, the apparent differences between the two year-classes in the Downs and, perhaps, in the Buchan stocks are hard to interpret. It is tempting, however, to point out that the percentage of immatures amongst 3-year-old fish was much higher in the 1957 than in the 1958 year-class.

2. Abundance estimates

The relative abundance of the different year-classes in the nursery areas is shown in Figure D IV 2. The 1960 year-class could be observed during the survey only as 0-group fish with a high abundance in the east, especially in the South-east area. In the autumn the 1959 year-class appears as 0-group fish in the South-east area, comprising about 80 per cent of the whole year-class. During the following winter and as I-group in the next summer the abundance decreases to a level of 25 per cent, while an increase in the West area is observed. The abundance of the 1958 year-class as I-group fish increased in the South-east area during the summer to the very high level of nearly 100 per cent; this means that at this age the total year-class passed through this area. In the following year the II-group fish of this year-class decreased, so that the population in the South-east went down to about 25 per cent with a corresponding increase in the West area. The 1957 year-class is observed only as II- and III-group at a level of 25-40 per cent in the South-east area. In the West area the population of this year-class is steady at 60 per cent.

Differences are observed between the 1958 and 1959 year-classes in the South-east area, in that the decrease in abundance starts in the 1959 year-class as 0-group fish in autumn, whilst the 1958 year-class has not then reached its highest level of relative abundance in this region. The reduction of the South-east portion of this year-class starts as I-group fish in autumn, thus a difference in time of nearly one year can be seen.

D V Adult year-class estimates

In section B I and II estimates of the abundance of the year-classes 1957 to 1960 in the immature phase have been given, based on catch-effort data and age compositions of the Bløden fishery and the 1960-61 research vessel surveys.

Section D III gives estimates of the Downs, Bank and Buchan components in each year-class of immature herring, derived from the results of the discriminant function analysis using the abundance estimates for the immature fish for the 1957, 1958 and 1960 year-classes.

After recruitment the year-classes sampled during the surveys joined the adult herring stocks in the North Sea. From the fisheries carried out on and in the vicinity of the spawning grounds, abundance estimates have been obtained for the individual year-classes of each stock. In principle, a comparison between year-class estimates in the immature and adult phase should be possible. Due to un-estimatable differences in effort and availability between the fisheries on the adult spawning stocks, it is not possible to compare the between-stock estimates of the strength of a given year-class. Within a single fishery for adult herring the estimates of successive year-classes are supposed to be comparable.

Year-class estimates are available for the Buchan stock from the Scottish drifter catches and for the Bank and Downs stocks from the Dutch herring-trawl fisheries (Table D V 1). Year-class strengths have been calculated as at 3 years of age (Cushing and Bridger 1966); however, it is not possible to put statistical levels of confidence on these means, with the available data.

Table D V 1 Abundance estimates for the 1957-60 year-classes as 3-year-old spawning fish; Downs and Bank stocks as thousands of fish caught per day trawling by a 500 b.h.p. vessel (maturity stage VI only), Buchan stock as crans per drifter haul

Year-class	Downs	Bank	Buchan
1957	9.6	2.0	4.4
1958	27.8	12.6	1.5
1959	1.0	0.7	1.6
1960	12.6	21.8	12.7

The impression from section B I, II and IV that the 1960 year-class was most abundant, probably followed by the 1958 year-class, and that the 1959 year-class was very weak, is in agreement with these general estimates of year-class strength in the adult stocks.

Now it is possible to obtain conversion factors between the different units of abundance in the three adult fisheries, making the following assumptions:

Let a be the proportion of a spawning stock taken by one unit of effort, b the survival rate between the immature and adult phase, and d the factor relating Y, the young herring abundance, with R, the absolute abundance of the young herring,

$$R = dY.$$

We then have:

$$R = Y/d = R_D + R_B + R_{Bu}.$$

Taking the catches per effort of 3-year-old fish in the spawning stocks we may write

$$Y/d = R = (C/f)_{D,3}/a_D b + (C/f)_{B,3}/a_B b + (C/f)_{Bu,3}/a_{Bu} b.$$

If we further suppose that the a's are independent of year-class and that b is independent of both year-class and stock, we found, writing X for d/ab:

$$Y = (C/f)_{D,3} X_D + (C/f)_{B,3} X_B + (C/f)_{Bu,3} X_{Bu},$$

in which X_D, X_B and X_{Bu} are the conversion factors between the three Pure Stocks.

The effort units on which Table D V 1 is based are nearly of the same magnitude, being the catch in thousands per day fishing (a cran is approximately a thousand herring). This means that the X's might be interpreted as estimates of the relative sizes of the spawning areas.

For each of the four year-classes, 1957-60, such an equation can be formulated, giving four equations with three unknowns (X_D, X_B and X_{Bu}). The values of year-class strengths in the young herring (Y) have been derived by combination of the values given in section B I, page 16, and section B II, page 20. These have been raised to the same level and the mean taken. Using the data of Table D V 1 for the values of the coefficients, we obtain the four equations:

1957 year-class	$52.5 = 9.6 X_D + 2.0 X_B + 4.4 X_{Bu}$
1958 year-class	$88.0 = 27.8 X_D + 12.6 X_B + 1.5 X_{Bu}$
1959 year-class	$16.5 = 1.0 X_D + 0.7 X_B + 1.6 X_{Bu}$
1960 year-class	$323.5 = 12.6 X_D + 21.8 X_B + 12.7 X_{Bu}$

These equations of young herring abundances (Y) on the adult catch per unit effort ($(C/f)_{D,3}$ etc.) represent four observations of a three-dimensional regression with zero intercept. Taking the Y's to have identical variances the estimates of the regression coefficients (X's) are:

$$X_D = -1.670, \quad X_B = 0.396, \quad X_{Bu} = 10.977.$$

For the corresponding analysis of variance we find:

Source	df	Sum of squares	Mean square	F
X_D, X_B and X_{Bu}	3	115 386.75	38 462.25	1 012
Remainder	1	38.00	38.00	
Total	4	115 424.75		

The standard deviations of the X's are:

$$S_{X_D} = 0.374, \quad S_{X_B} = 0.853, \quad S_{X_{Bu}} = 1.156.$$

The conversion factors found are not well suited for further use because of the negative X_D. The analysis of variance, however, shows a relation between the Y's and (C/f)'s, and the standard deviations show that the figures are not inconsistent with a hypothesis assuming the Downs conversion factor to be small as compared with the Bank and Buchan factors. It is, however, unrealistic to take the variances of the Y's to be identical. It is perhaps more to the point to use identical relative standard deviations (see Appendix 3). This is approximately effected if we use the following relative weights, W, taking the value for Y_{1959} as unity

$$\left[W(Y) = \frac{1}{(Y/16.5)^2}\right];$$

then $W(Y_{1957}) = 0.099$, $W(Y_{1958}) = 0.035$, and $W(Y_{1960}) = 0.003$.

The regression estimates now become:

$$X_D = 0.916, \quad X_B = 8.355, \quad X_{Bu} = 8.769$$

with the analysis of variance

Source	df	Sum of squares	Mean square	F
X_D, X_B and X_{Bu}	3	1 114.45	371.48	23.7
Remainder	1	15.67	15.67	
Total	4	1 130.12		

and the following standard deviations:

$$S_{X_D} = 1.588, \quad S_{X_B} = 3.454, \quad S_{X_{Bu}} = 2.146.$$

The X-solutions are very similar to the former set, but the relation between Y's and (C/f)'s has faded away even if the hypothesis on the size of the Downs conversion factor is not contradicted by the figures. The analysis gives an estimate of the relative standard deviation of Y, viz: $\sqrt{15.65}/16.5 = 24$ per cent, a figure that compares well with the figures shown in Appendix 3. The difference between the two regressions is that this regression is very much influenced by Y_{1959}, and this can perhaps be taken as suggesting that the 1959 year-class was peculiar. The main difficulty with this analysis is that the variance is based on one degree of freedom. The remedy of this would have been to use a long series of year-classes.

Another approach would be to select three of the four equations in all possible ways (four) and solve them. This gives the following solutions (Table D V 2):

Table D V 2 Conversion factors (X_D, X_B, X_{Bu}) obtained by solving three equations in four possible combinations (see text)

Year-classes used	X_D	X_B	X_{Bu}
1957, 1958, 1959	0.998	3.828	8.014
1957, 1958, 1960	-1.616	9.207	11.272
1957, 1959, 1960	0.913	11.591	4.671
1958, 1959, 1960	-2.946	12.698	6.598

Of these four solutions two again give a small negative conversion estimate for the Downs stock, the other two giving positive conversion estimates for all three stocks. However, all estimates for the Downs stock could well indicate a small conversion factor for this stock as compared with those for the other two stocks, keeping in mind that with only one degree of freedom the four solutions are equally good estimates of the three conversion factors.

The two solutions which give positive estimates for X for all three stocks can be used in a further exercise by applying the estimated conversion factors to the adult year-class estimates ($(C/f)_{D,3}$ etc.). Hereby we obtain comparable abundance estimates between year-classes and stocks (Table D V 3). These figures indicate, on average, a large Buchan and Bank and a much smaller Downs recruit population in these four year-classes. The three year-classes on which the solutions are based will of course fit the respective equations exactly. A complete fit for the remaining year-class would have been obtained if the 1960 year-class was overestimated as immatures (1957, 1958, 1959 equations) or if the 1958 year-class was underestimated as immature fish (1957, 1959, 1960 equations).

Table D V 3 Estimates of the abundance of the 1957-60 year-classes in the Downs, Bank and Buchan stocks, obtained by applying the conversion factors of Table D V 2 (1957, 1958, 1959 and 1957, 1959, 1960 solutions) to the estimates of Table D V 1 (see text)

Year-class	Downs	Bank	Buchan	Total	Downs	Bank	Buchan	Total
	X-values 1957, 1958, 1959				X-values 1957, 1959, 1960			
1957	9.6	7.7	35.3	52.6	8.8	23.2	20.6	52.6
1958	27.7	48.2	12.0	87.9	25.4	146.0	7.0	178.4
1959	1.0	2.7	12.8	16.5	0.9	8.1	7.5	16.5
1960	12.6	83.5	101.8	197.9	11.5	252.7	59.3	323.5

There are no reasons to suppose that the 1958 year-class was underestimated in the surveys and by the Bløden fishery in 1960-61. The 1960 year-class, however, could have been overestimated, since the estimate of this year-class was based on only one comparison, i.e. with the 1959 year-class in the autumn as 0-group fish, and the 0-group fish of the 1960 year-class (14.6 cm) were slightly larger than those of the 1959 year-class (14.3 cm). It is likely that because of this difference in size, the 1960 year-class had a more advanced recruitment from the coastal nurseries to the open sea, so that the year-class was overestimated relative to the other year-classes. This impression is supported by a comparison of the abundance of the four year-classes as I-group fish (average spring and autumn) in the Bløden fishery in the years 1959-62 (ICES 1969), which indicates that the relative abundance of the 1960 year-class was overestimated by about 40 per cent in the Y values used in our equations. Such an estimate (40 per cent) tallies well with the results of the 1957, 1958, 1959 equations (Table D V 3). Moreover, the conversion factors (X's) found by using these equations appear to be reasonable: the lowest X value is found for the smallest area (Downs), and the largest one for the large Buchan area.

Finally, the estimates of the recruit strengths to the three stocks, as shown above, may be compared with the results of the discriminant function analysis. In Table D III 9 the strengths of recruitment to the three stocks in two year-classes (1957 and 1958), as derived by the discriminant analysis, using the abundance raising factors, are given.

A comparison shows that the recruitment estimates to the Buchan and Bank stocks, based on the discriminant function analysis, bear some resemblance to the results of the method applied in this section. Recruitment estimates to the Downs stock, however, tend to be higher by the discriminant function analysis than any derived by this method.

The results of the discrimination for the 1960 year-class given in Table D III 7 compare better with the results of the present method, except when the abundance indices including the ship's power factors (method A) are used.

D VI Otoliths

The difficulty of using otolith data to decide the composition of a mixed population of these three stocks is that, in general, Bank fish have otolith characteristics intermediate between those of Downs and Buchan fish; therefore, where the otolith type of the mixed stock is intermediate it is not possible, on otolith typing alone, to say whether one is dealing with a pure Bank stock or a mixture of Downs and Buchan fish. From the data in Table C IV 3 the high proportion of "wides" in the total North Sea probably indicates a low proportion of Buchan fish in the 1957 and 1958 year-classes. These relatively high proportions of "wides" might further be interpreted as suggesting that the Downs stock is the dominant stock in the Bank/Downs mixture.

The other feature of these data given in Table C IV 3 is that, in general, where the otolith analysis is based on an adequate number of fish the percentage of "narrow" otolith types is lowest in the south-east in all seasons, while the proportion tends to be highest in the western area. In fact, in the south-east the percentage of "narrows" is generally lower than in any of the Pure Stocks. As pointed out by Parrish and Sharman (1958) wide-type otoliths generally have a low modal l_1. These data therefore suggest that not only is the south-eastern area principally a nursery for the Downs component of the stock, but also it is mainly the "wide" component of that stock which is to be found there. In view of the fact that the percentage of "wides" is usually higher than that in any of the Pure Stocks, there is a suggestion of some length segregation. However, as there is no way of testing the significance of the differences in the proportions of "wides" too much weight should not be given to this suggestion.

Otolith typing cannot be used to give an independent quantitative breakdown (into the component stocks) of the young herring population caught on these surveys, because there are two independent variables in the stock composition but only one in the otolith characters. The most critical analysis that can be made with these data is to calculate, from the allocation to stocks made by the discriminant function analysis, for each year-class on each survey, what the percentage of otolith types would be in the mixed population and compare this with the percentage found for that year-class on that survey in the otolith analysis. Unfortunately, owing to the failure of the Pure Stock characters of the 1959 year-class, no test can be made of the success of the discriminant analysis. Likewise, in the case of the 1960 year-class no test is available, because the fish are 0-group herring without the winter ring for typing.

For the 1957 and 1958 year-classes the calculations were made for each estimate to check the relative reliabilities of stock abundances based on the discriminations using $l_1/VS/K_2$ and VS/K_2, and using total young herring abundances incorporating (1) correction factors for the Bløden fishery and ship power factors, (2) corrections for the Bløden fishery but not for ship power factors, and (3) with no corrections at all.

The results of these calculations are given in Table D VI 1, together with the corresponding data for the otolith proportions of the young herring sampled on the surveys as given in Table C IV 3. From these data it would seem clear that the discrimination based on $l_1/VS/K_2$ gives results more compatible with those of the survey material than those based on a VS/K_2 discrimination.

Using the same criterion the various abundance indices listed would not seem to have any very marked effect on the goodness of fit. The abundance indices based on a Bløden correction factor + ship power factors, and with no weighting at all, are perhaps slightly superior to that with only a correction for the Bløden fishery. But certainly the first two cannot be separated on this criterion.

Using the $l_1/VS/K_2$ discrimination and abundance indices either unweighted or weighted by both power factors and a Bløden correction it would appear that the agreement is reasonably close in the case of the 1957 year-class on all the surveys given in Table D VI 1. In the 1958 year-class similar proportions to those in the mixed fishery occur only in the 1960 autumn survey, for it should be appreciated that very large changes in the stock proportions are needed to give appreciable differences in percentage otolith type. For the 1958 year-class in the spring and autumn of 1961 these data would suggest that the Downs component has been underestimated. Thus, from the otoliths it would appear that the discriminant function analysis, using $l_1/VS/K_2$, gave fairly reasonable results for the 1957 year-class, and in the case of the 1958 year-class underestimated the Downs component somewhat. However, there is no statistical method for testing the significances of the differences in the otolith type allocations in the absence of variance estimates. Moreover it should be remembered that the otolith typing used for the immatures was obtained by applying corrections (see section C IV) which may have introduced errors.

Table D VI 1 Comparison of the separation of otolith types by the discriminant function stock allocation, using the $l_1/VS/K_2$ and VS/K_2 discriminations, and applying three sets of abundance raising factors, with those found by the otolith analysis (Table C IV 3). All figures shown are percentages

Year-class	Survey	Abundance estimates with power factor + Bløden adjustment						Abundance estimates without power factors, but with Bløden adjustment						Abundance estimates without any correction						Survey data from Table C IV 3		
		$l_1/VS/K_2$			VS/K_2			$l_1/VS/K_2$			VS/K_2			$l_1/VS/K_2$			VS/K_2					
		narrow	wide	?	narrow	wide	?	narrow	wide	?	narrow	wide	?	narrow	wide	?	narrow	wide	?	narrow	wide	?
1957	Spring 1960	61.3	33.2	5.5	64.4	29.3	6.3	60.8	33.7	5.5	66.3	27.2	6.5	61.5	33.0	5.5	66.4	27.3	6.3	60.1	36.4	3.5
1957	Autumn 1960	52.8	40.5	6.8	63.1	31.6	5.3	50.4	42.8	6.8	62.3	32.6	5.2	50.7	42.3	7.0	62.2	32.6	5.2	54.8	42.2	3.1
1957	Spring 1961	66.3	24.6	9.1	67.0	27.4	5.6	63.3	24.5	9.1	67.2	27.2	5.6	66.2	24.6	9.3	69.0	25.4	5.7	71.5	24.1	4.4
1958	Autumn 1960	51.9	46.8	1.3	61.4	36.9	1.7	51.4	47.0	1.3	61.3	37.1	1.7	51.8	47.0	1.3	61.3	31.0	1.7	51.9	46.5	1.6
1958	Spring 1961	56.9	41.6	1.5	69.7	28.6	1.7	56.8	41.7	1.5	69.6	28.7	1.7	56.6	41.9	1.5	70.3	27.9	1.7	48.8	47.7	3.5
1958	Autumn 1961	61.0	37.4	1.6	65.5	32.9	1.6	61.4	37.0	1.6	64.8	33.5	1.7	62.2	36.2	1.6	65.5	33.0	1.6	57.0	42.5	0.5

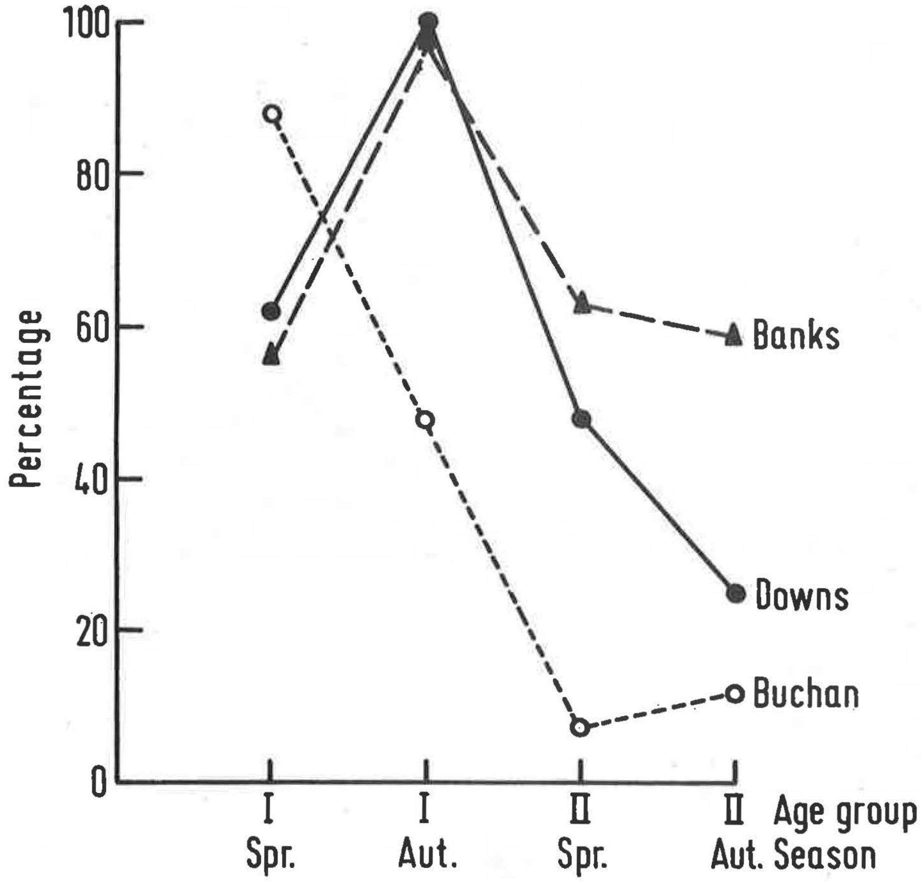


Figure D IV 1 **Seasonal changes in mean percentage abundances of Downs, Bank and Buchan stocks (from Table D IV 1).**

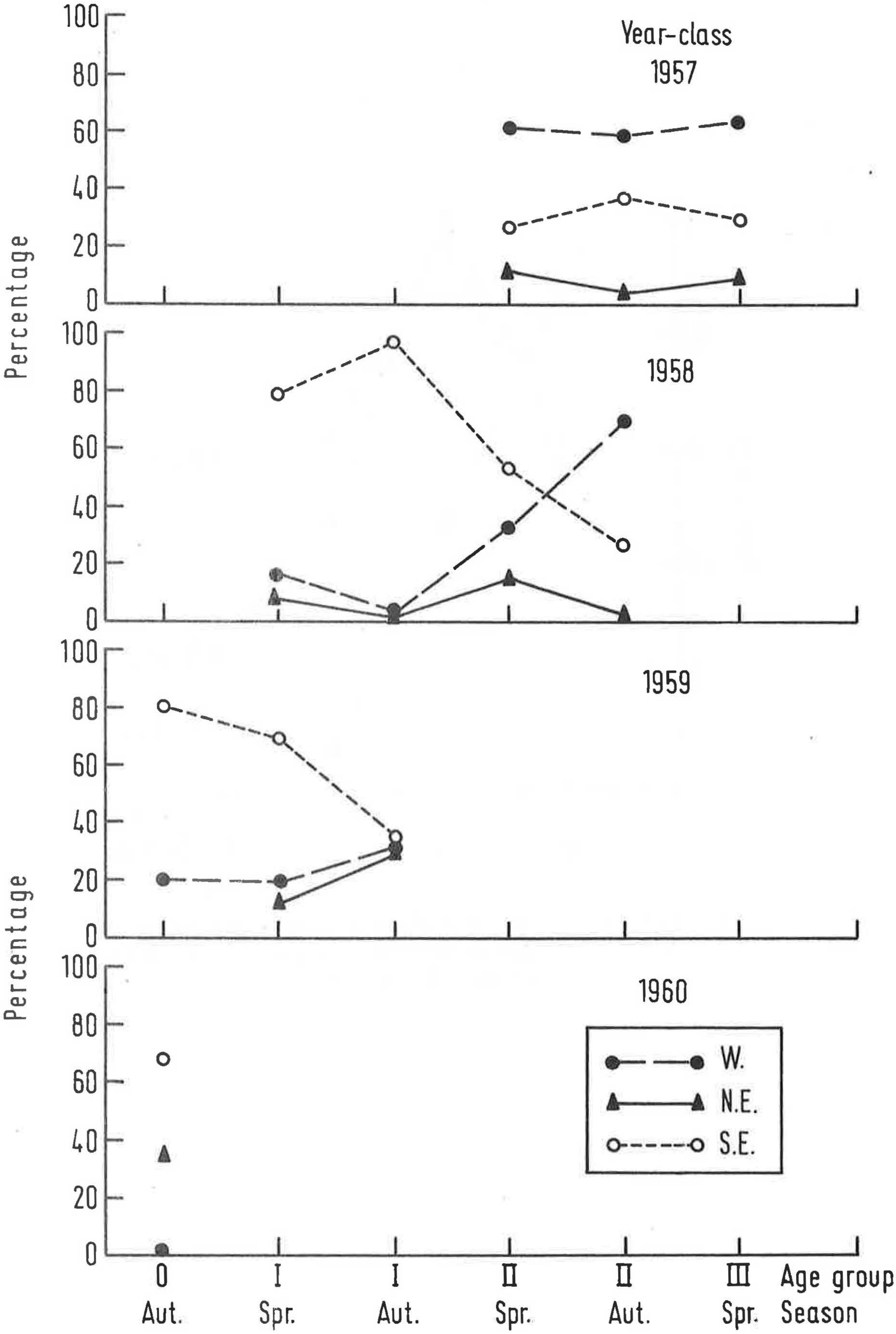


Figure D IV 2 Relative abundances of the different year-classes in the areas West, North-east and South-east.

E DISCUSSION

As pointed out in the introduction the surveys were initially carried out to identify the main centres of abundance of pre-recruit herring in the North Sea and to follow how these changed with increasing age of the fish. In general these aims were attained, although with four surveys it must be admitted that only a limited indication is obtained of the variability in distribution from year to year.

Main centres of distribution of immature herring were found in the central North Sea - to the east and west of the Dogger Bank. Lesser ones were located in the Skagerak, Moray Firth, and the eastern part of the Southern Bight. Older immatures (II-group) were also found in the Egersund area. The distribution of immatures in spring was more extensive than in autumn, when they were largely confined to the area around the Dogger. With increasing age the immature herring tended to migrate in a north-westerly direction. There was some indication of differences in migration between year-classes which might be related to differences in stock composition of the immature year-classes (see section D IV). There was a consistent tendency for the smaller fish of a year-class to be found in the area of the German Bight and off the Danish coast, and the larger fish to the west and north of the Dogger Bank.

In autumn the distribution of the immature herring showed some relation to areas of sharp changes in bottom temperatures. The size of the change rather than any specific temperature seemed to be the controlling factor. In spring there was an indication that the distribution of juvenile herring from the Skagerak nursery into the north-eastern North Sea to as far north as Egersund was related to the spread of cold, low-salinity water into that area.

The relative strengths of the year-classes sampled could only be estimated rather roughly from the survey data. The results, however, were consistent with those estimated from catch and effort data of the Bløden fishery and were compatible with recruitment strengths to the adult stocks.

It was when one attempted to separate the three North Sea stocks within the total body of immatures that major difficulties arose. For this purpose use was made of the discriminant function technique. Fukuhara (1960) used this technique to determine the proportions of south-western Kamchatkan and western Alaskan red salmon in high-seas catches. The discriminant function was composed from seven meristic characters which had been chosen on the basis of certain a priori information. Confidence limits were obtained for the classification of western Alaskan red salmon which were dependent on sample size. Samples of less than 25 fish gave estimates with limits ± 31 to ± 46 per cent. For samples of, say, 100 or more, limits better than ± 20 per cent were obtained. The technique can thus be a very powerful tool in stock separation within a mixed fishery.

In the young herring clear results are more difficult to demonstrate. Some of the difficulties were connected with the definition of the Pure Stocks of which the immatures encountered during the surveys were thought to be a mixture. Thus a question which had to be taken into account in considering the validity of the discrimination was whether each of the three Pure Stocks is in reality a unit stock in the strict definition of the term or an amalgam of two or more stocks which differ in one or more of the characters considered. The Downs stock, for example, comprises fish which spawn at Sandettie and in the eastern Channel, and it has been shown that there are consistent differences in mean vertebral counts between Sandettié and eastern Channel spawners (le Gall 1936, Zijlstra 1958, Krefft 1963, ICES 1965). Similarly Zijlstra (1963) has shown for spawners in the central North Sea that there is a consistent trend in mean vertebral count and length (l_1) with time during the spawning season. In the Buchan area the spawning grounds are known to be very extensive and here too there seems to be scope for considerable differences in meristic characters between the fish spawning on different grounds within the area of this stock. Data available for the 1957, 1958 and 1959 year-classes were shown in Table C III 2, in which are indicated differences in VS and K_2 between the fish on the various spawning grounds of the Buchan Pure Stock.

Whether this means that in the North Sea there are not, in reality, three spawning stocks but a considerable number of completely independent spawning stocks is not a point which can be fully discussed here. But whether the latter is true or not, the fact remains that in time or in area, within the three stocks, one can get considerable differences in meristic characters and l_1 and it shows the difficulty of establishing reliable mean stock characters.

A successful discrimination of the stock components in a mixture is dependent on obtaining representative Pure Stock characters. Though the sampling of the Pure Stocks covered the complete range in space and time of the commercial fisheries on each spawning stock and is therefore thought to be fairly representative, a great accuracy in the discriminations cannot be expected.

A second problem, related to the Pure Stocks, was that our analysis assumed the presence of only three Pure Stocks in the immature herring. As has been pointed out, in the 1959 year-class there is good reason to think that this was not true. Figures C III 1 and C III 2 show that for the 1959 year-class it is impossible to accept from the VS/K_2 and the l_1/K_2 plots that only representatives of the Buchan, Bank and Downs stocks were included in the young herring population. However, if one accepts that representatives of a stock with characters similar to those of the Minch autumn-spawning stock formed a considerable proportion of the immature fish of the 1959 year-class, which was extremely weak in the three North Sea stocks, the distributions of l_1, K_2 and VS are explicable. A discrimination attempted on this year-class using l_1/VS/K_2 and VS/K_2 gave nonsensical results.

Other difficulties were connected with the characters used in the analysis. Only three characters, VS, K_2 and l_1, were available as stock characters for the discriminant function, and it was clear that of these l_1 had the greatest discriminant power. However, l_1 was also the character most poorly sampled in the young herring because of the difficulty of getting scales from trawled herring. The result was that a standardized l_1 had to be used both for the young fish and for the adult Pure Stocks.

As will be seen from Table C II 8 the standardized mean l_1 for the Buchan stock is consistently higher than the observed one; for the Downs stock this standardized mean l_1 varies little from the observed one. This could have introduced some bias into the discriminations using l_1 and might tend to overestimate the Downs component.

There seems no reason to suspect a bias in the VS/K_2 discrimination, since these meristic characters are free of any length bias. From a comparison of the $l_1/VS/K_2$ and the VS/K_2 discriminations, shown in Table D II 2, it appears that the introduction of l_1 has shifted the splitting lines in the discrimination towards the Buchan centre, giving higher proportions of Downs fish.

The discriminant power of VS/K_2 is, however, relatively small and the relatively unbiased VS/K_2 discriminations had a much higher variance than the probably biased $l_1/VS/K_2$ or l_1/K_2 discriminations (Tables D II 1.1-4).

A further source of difficulties was caused by using raising factors to convert the allocations by areas into a total North Sea allocation, which introduced high variances and a possible bias. For two out of three sets of raising factors the variances could be estimated and were found to be high, as shown in Appendix 3. Consequently, these high variances on the raising factors introduced a high variance on the total North Sea allocation, as is indicated in Tables D III 1-7. Besides, if the availability of the fish to trawling varies between areas, this could also introduce a bias in the total allocation to stocks, since the analysis suggested that the stock proportions varied between areas. Although no firm evidence can be produced, there was a feeling that immature herring were more available to capture by trawl in the South-east area than in the other areas. As Downs fish are probably the dominant component in the South-east area this again could lead to overestimation of the Downs stock in the total North Sea allocation.

The conclusion drawn about the migrations of the immatures of the three stocks (section D IV) will probably not be greatly affected by such biases. The biases will affect the amplitude of the curves of Figure D IV 1 rather than the form.

The relative year-class sizes both within and between stocks in the total North Sea allocation will be affected by such biases. The otolith material, on the other hand, tended to suggest an underestimation of the Downs stock, by a discriminant function, at least in one year-class (1958). However, because of the methods used to correct the immature otolith typing, described in section C III, not much reliance can be placed on the otolith check. The attempts made to compare stock sizes by year-classes from the survey material with the data on year-class strength in the adult fisheries (section D V) showed serious discrepancies, which could be attributable to biases in the survey material. The method described in section D V indicated in most cases larger Buchan-Bank components than the discriminant function analysis, which complies well with the direction the biases are thought to take.

F CONCLUSIONS

1. Main herring nurseries in the North Sea were found to the east and west of the Dogger Bank, with subsidiary ones in the Skagerak, Moray Firth and the Southern Bight.

2. In the main nurseries the age of the immatures tended to increase from south-east to north-west. In addition, the mean length per age-group increased in the same direction.

3. Relations found between fish distribution and hydrographic features were somewhat tenuous.

4. Year-class variations in the abundance of the immature herring were in general compatible with those in the adult North Sea stocks.

5. In the 1959 year-class, for which the discriminant function analysis failed, an enquiry revealed that there were indications of the presence in the North Sea of juvenile herring having VS and K_2 characters similar to those of Minch autumn spawners.

6. The discriminant function analyses indicated a similar pattern but a different timing in the migration of juveniles of the three North Sea stocks (Downs, Bank and Buchan) through the nurseries. Immatures of the Bank and Downs stocks were largely concentrated in the south-eastern North Sea in the autumn of their second year (I-group), whereas the Buchan immatures tended to have their major concentrations in that area at least six months earlier.

7. The discriminant analysis for the 1957 and 1958 year-classes, which were the only ones on which any reliance could be placed, indicated the proportion of Downs fish to be the largest. The Bank proportion would appear to be the smallest in the 1957 year-class and that of the Buchan in the 1958 year-class. Because of probable biases in the analysis and in the abundance weighting factors it is likely that the Downs component was overestimated in these assessments. This impression is supported by the assessments of relative year-class strength between stocks made in section D V.

G ACKNOWLEDGEMENTS

With a cooperative work of this nature it is difficult to mention all the help given in the preparation of the report. The biological material comprised data from some 35 000 fish each described by sixteen characters. The very extent of the data is a recognition of the work of the technicians in the various national laboratories.

The operation of transferring these data to paper tape and the verification and correction of these tapes for use in the computer was of considerable magnitude. Mrs. Nancy Berry of the Fisheries Laboratory, Lowestoft supervised the whole of this operation and in addition wrote the many versions of the computer programs used in the long analyses of the data. The extent and development of these programs is outlined in Appendix 2. The Working Group wishes to place on record its indebtedness to Mrs. Berry and also the staff of the MAFF 803 Unit for their patience in dealing with the many changes which were made during the development of the analysis.

The draft report of 1968 and this present report were both prepared and typed in the Fisheries Laboratory, Lowestoft. Miss Rose Bedford has painstakingly proof-read both reports and we wish to thank her for her conscientious work on them.

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Appendix 1 AN ANALYSIS OF COMPARATIVE AGE, l_1, l_2, AND OTOLITH TYPE DATA

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The data used in this analysis resulted from an examination of six North Sea herring samples by Danish, German, English, Dutch, and Scottish workers in preparation for the meeting of the ICES North Sea Herring Methods Working Group.

Since the results of the examinations were circulated to the participants in advance of the meeting, the full details are not presented here; only extracts are given in App. 1, Tables 1-3.

1. l_1 measurements

In the calculation only fish with all five l_1 determinations are utilized, because the high number of missing values would make a statistical treatment of the whole material very time-consuming and complicated. In App. 1, Table 1 the data used in the analysis are given. A few additional values have been discarded, as it was obvious that different rings had been used for the l_1 determinations in the five countries. The following mathematical model has been used: the l_1 measurements are supposed to have the following form:

$$l_{1,i,j,k} = \lambda_{1,k} + f_{i,k} + c_{j,k} + \epsilon_{i,j,k}, \quad \text{(i)}$$

where (1) λ, f and c are constants, (2) i refers to the individual fish, (3) j refers to the country, (4) k refers to the area (the six samples consist of two from each of three areas), and (5) the ϵ's are stochastic components.

This model is a so-called two-way classification. If it is demanded that $\Sigma f = \Sigma c = 0$, $\lambda_{1,k}$ will be the mean l_1 for the area k.

It is further supposed that (6) the ϵ's are all independent and normally distributed (0, σ_k).

The sum of squares $\Sigma l_{1,i,j,k}^2$ for an area can now be split up in the following way:

$$\Sigma l_{1,i,j,k}^2 = \text{(contribution from the mean)} + \text{(contribution from the f's)} + \text{(contribution from the c's)} + \text{remainder}$$

or in a specified form

Contribution from the mean $A = \frac{(\Sigma 1_{1,i,j,k})^2}{r} \cdot s$

Contribution from the f's $B = \Sigma^r \frac{(\Sigma^s 1_{1,i,j,k})^2}{s} - A$

Contribution from the c's $C = \Sigma_{j=1}^{s} \frac{(\Sigma_{i=1}^{r} 1_{1,i,j,k})^2}{r} - A$

Remainder $D = \text{Total} - (A + B + C)$

Total $\Sigma 1^2_{1,i,j,k}$

(r = number of fish, s = number of countries).

The expectations and degrees of freedom of the sums of squares are:

	expectation	df (degrees of freedom)
Contribution from the mean	$r \cdot s \cdot \lambda^2_{1,k} + \sigma^2_k$	1
Contribution from the f's	$(r-1)\sigma^2_k + s(r-1)\sigma^2_f$	r -1
Contribution from the c's	$(s-1)\sigma^2_k + r(s-1)\sigma^2_c$	s - 1
Remainder	$(r-1)\ (s-1)\ \sigma^2_k$	(r-1) (s-1)

where $\sigma^2_f = \frac{\Sigma f^2}{r - 1}$ and $\sigma^2_c = \frac{\Sigma c^2}{s - 1}$.

The expectations of the mean squares are:

Mean	$\sigma^2_k + r\, s\, \lambda^2_{1,k}$
f	$\sigma^2_k + s\, \sigma^2_f$
c	$\sigma^2_k + r\, \sigma^2_c$
Remainder	σ^2_k .

The hypothesis $c_1 = c_2 = \dots\dots\dots\dots = c_s = 0$ can now be tested by means of

$$v^2 = \frac{c \text{ mean square}}{\text{Remainder mean square}}$$

which, according to the hypothesis, is v^2 distributed with $s - 1$ and $(r-1)(s-1)$ degrees of freedom, and this test is independent of the values of the f's. The proposed model is not fulfilled for all the data in App. 1, Table 1 as the Danish measurements are to the half centimetre below, whereas all other measurements are to the nearest millimetre. The Danish measurements are therefore excluded from the analysis of variance shown here:

1. Area 1 (Samples 14 E A 61 and 18 E A 61)

Contribution from	df	Sum of squares	Mean square	v^2
Mean	1	2 042 362.06		
f's	39	71 751.69		
c's	3	471.52	157.17	15.03
Remainder	117	1 223.73	10.459	
Total	160	2 115 809.00		

2. Area 2 (Samples H 43 + H 44)

Contribution from	df	Sum of squares	Mean square	v^2
Mean	1	1 305 224.13		
f's	16	43 487.12		
c's	3	133.22	44.407	7.82
Remainder	48	272.53	5.6777	
Total	68	1 349 117.00		

3. Area 3 (Samples FR 22 July 1958 and FR 16 August 1958)

Contribution from	df	Sum of squares	Mean square	v^2
Mean	1	3 392 957.61		
f's	44	101 904.64		
c's	3	296.59	98.863	9.06
Remainder	132	1 440.16	10.910	
Total	180	3 496 599.00		

The three v^2 values are all highly significant, and the hypothesis $c_1 = c_2 = c_3 = c_4$ therefore is strongly rejected.

The next table shows the c values for the three localities:

	EA	H	FR
c_1 (Germany)	+0.07	-0.13	-0.31
c_2 (England)	-2.50	-2.01	-1.76
c_3 (Netherlands)	+2.34	+1.93	+1.80
c_4 (Scotland)	+0.10	+0.22	+0.35

The c values are very consistent and for the three variances Bartlett's Test gives $\chi^2 \approx 7.01$ with two degrees of freedom, which gives $5\% > p > 2.5\%$. It is in this way reasonable to pool the data. If we do so we get a new analysis of variance:

Contribution from	df	Sum of squares	Mean square	v^2
Mean	1	6 681 344.41		
f's	101	276 342.84		
c's	3	877.09	292.36	29.92
Remainder	303	2 960.66	9.7712	
Total	408	6 961 525.00		

and the following c values:

c_1 (Germany)	-0.17
c_2 (England)	-2.10
c_3 (Netherlands)	+2.03
c_4 (Scotland)	+0.23

The difference between two c's has the variance

$$\frac{2\sigma^2}{102} \approx 2 \times \frac{0.7712}{102} = 0.19159 = (0.43771)^2 ,$$

and confidence limits can now be calculated for the differences:

	Δc	95% confidence limits
Germany-England	+1.93	+1.07, +2.79
Germany-Netherlands	-2.20	-3.06, -1.34
Germany-Scotland	-0.40	-1.26, +1.46
England-Netherlands	-4.13	-4.99, -3.27
England-Scotland	-2.33	-3.19, -1.47
Netherlands-Scotland	+1.80	+0.94, +2.66

If we calculate c_0 (Denmark) and correct for measuring to the half centimetre below we get:

c_0 (Denmark)	+1.07
c_1 (Germany)	-0.44
c_2 (England)	-2.37
c_3 (Netherlands)	+1.76
c_4 (Scotland)	-0.04

Denmark-Germany	+1.51
Denmark-England	+3.44
Denmark-Netherlands	-0.69
Denmark-Scotland	+1.11

2. l_2 measurements

In App. 1, Table 2 are given the l_2 measurements in the same way as the l_1 measurements in App. 1, Table 1 and we get the following analysis of variance:

Area 1 (Samples 14 E A 61 + 18 E A 61)

Contribution from	df	Sum of squares	Mean square	v^2
Mean	1	5 674 597.30		
f's	36	28 805.20		
c's	3	356.59	118.86	10.76
Remainder	108	1 192.91	11.046	
Total	148	5 699 952.00		

Area 2 (Samples H 43 and H 44)

Contribution from	df	Sum of squares	Mean square	v^2
Mean	1	3 005 455.64		
f's	15	25 947.61		
c's	3	36.92	12.307	1.45
Remainder	45	382.83	8.5073	
Total	64	3 031 823.00		

Area 3 (Samples FR 22 July 1958 and FR 16 August 1958)

Contribution from	df	Sum of squares	Mean square	v^2
Mean	1	7 892 327.53		
f's	42	61 157.47		
c's	3	262.47	87.490	18.60
Remainder	126	592.53	4.7026	
Total	172	7 954 340.00		

The v^2 values are highly significant for Areas 1 and 3 but not significant for Area 2. A calculation of the c's gives:

	Area 1	Area 2	Area 3
c_1 (Germany)	-0.78	-0.95	-1.51
c_2 (England)	-1.59	-0.14	-0.88
c_3 (Netherlands)	+2.54	+1.18	+1.49
c_4 (Scotland)	-0.16	-0.08	+0.91

Even if these figures look less consistent than the corresponding l_1 values, there is nevertheless satisfactory agreement. The variances, on the other hand, are not in agreement, as Bartlett's Test gives $\chi^2 \approx 21.13$ with 2 degrees of freedom and $P \ll 0.05$ per cent. It is therefore not wise to pool the data but we can find mean $(c_i - c_j)$'s by using the weights $r_k : 2\sigma_k^2$, which are the reciprocals of the variance of $c_i - c_j$. This procedure gives, taking the corrected Danish data into account:

	$(c_i - c_i)$	95% confidence interval
Denmark-Germany	+0.62	(-0.13, +1.37)
Denmark-England	+0.32	(-0.43, +1.07)
Denmark-Netherlands	-2.35	(-3.10, -1.60)
Denmark-Scotland	-1.19	(-1.94, -0.44)
Germany-England	-0.30	-1.05, +0.45
Germany-Netherlands	-2.97	-3.72, -2.22
Germany-Scotland	-1.81	-2.56, -1.06
England-Netherlands	-2.67	-3.42, -1.92
England-Scotland	-1.51	-2.26, -0.76
Netherlands-Scotland	+1.16	+0.41, +1.91

The confidence interval is found as 2 . s, where $1 : s^2 = \Sigma r_k : 2 s_k^2$. This procedure is not quite correct for the Danish figures, as mentioned before, but the approximation is reasonably good.

For the c values we get:

c_0 (Denmark)	-0.52
c_1 (Germany)	-1.14
c_2 (England)	-0.84
c_3 (Netherlands)	+1.83
c_4 (Scotland)	+0.67 .

Discussion

From the above analysis of variance it is quite clear that there exist highly significant differences between countries. The differences are consistent for the l_1 and l_2 measurements respectively. For comparing the l_1 and l_2 measurements App. 1, Figure 1 has been drawn, which gives the $(c_i - c_j)$'s and the confidence limits. As the fish lengths were given, one should expect differences between l_1 and l_2 measurements, if l_1 differences between countries exist, but the sort of differences to be expected would be a sort of similarity, the l_2 countries'difference values being the smaller ones. The l_2 values are the smaller ones, but the picture is not one of similarity. There are, in fact, specific l_1 differences and specific l_2 differences. As regards the variances, which are estimates of the measuring error, they are of the order of magnitude of 10 $mm^2 \approx (3\ mm)^2$ and compare well with the estimates found by Burd (personal communication), but it has to be borne in mind that only the best scales have been used in the calculations, so that the variance found is certainly an underestimate of the true measuring error. (Unfortunately the Netherlands data had a systematic error due to faulty equipment when these analyses were made. This could account for the high positive values reported here. Editor, 1968.)

3. Age determination

For the scale and otolith readings the following model is being used:

If a is the correct reading of a scale (otolith) there is a probability P_i' for determining the age as a-1, P'' for a+1, and $1-P_i' - P_i''$ for a. Here i refers to countries and it is supposed that P is independent of age.

A reading x_{ijk} can then be written as:

$$x_{ijk} = a_{jk} + \epsilon_{ijk}$$

where a_{jk} is the correct age of the j'th fish from sample number k, and ϵ is a discrete stochastic variable with mean $P_i'' - P_i'$ and variance $P_i' + P_i'' - (P_i'' - P_i')^2$ which approximates to $P_i' + P_i''$, if $P_i'' - P_i'$ is small.

If n_k fish from sample k have been used for age determination, the estimated mean age will be:

$$\bar{x}_{ik} = \frac{\Sigma a_{jk}}{n_k} + \frac{\Sigma \epsilon_{ijk}}{n_k} = \frac{\Sigma a_{jk}}{n_k} + P_i'' - P_i' + \frac{\Sigma \eta_{ijk}}{n_k}$$

where η_{ijk} has mean 0 and variance $P_i' + P_i''$ (app.). If all n_k's are equal, all $(P_i' + P_i'')$ are equal, and $\Sigma(P_i'' - P_i') = 0$, then the mean ages for sample number k can be written as:

$$\bar{x}_{ik} = \bar{a} + S_k + (P_i'' - P_i') + \rho_{ik}$$

where $\bar{a}$ is the mean age of all fishes, S_k a sample difference with $\Sigma S_k = 0$, and ρ_{ik} is a stochastic variable approximately normally distributed

$$(0, \sqrt{\frac{P' + P''}{n_k}}) \qquad \text{(the central limit theorem)}$$

In the following analysis only fish which have got both a scale and an otolith age reading have been used. The numbers of these fish are not constant for the six samples but very nearly so (the numbers are in fact 42, 46, 47, 44, 45 and 49). The proposed model will in this way still be correct if n_k is replaced by the mean number of fish with both scale and otolith readings. In App. 1, Table 3 the mean ages for the six samples are given, and the above model is exactly analogous to the model used for the l_1 and l_2 measurements. The data give the following analysis of variance:

Scale readings

Contribution from	df	Sum of squares	Mean square	v^2
Mean	1	282.467631		
S's	5	9.779251		
(P' - P'')'s	4	0.016558	0.0041395	4.08
Remainder	20	0.020304	0.0010152	
Total	30	292.280744		

Otolith readings

Contribution from	df	Sum of squares	Mean square	v^2
Mean	1	285.559942		
S's	5	10.248069		
(P' + P")'s	4	0.003170	0.0007925	1.26
Remainder	20	0.012604	0.0006302	
Total	30	295.823785		

The v^2 value is significant (2.5% > P > 1%) for the scales but not for the otoliths. This means that differences between countries probably exist for the scale readings but not for the otolith readings. The variances (remainder mean square) are very nearly the same for otoliths and scales. The assumption $\Sigma(P'' - P') = 0$ is equivalent to the assumption that the mean of all countries has the correct age as expectation, and from App. 1, Table 2 we get for scales:

	P" - P'
Denmark	+0.0033
Germany	+0.0258
England	-0.0388
Netherlands	+0.0210
Scotland	-0.0115

and as $P'' + P' = 45.5\ \sigma \approx 45.5 \times 0.0010152 = 0.0462$ we get

	P' (%)	1-P'-P" (%)	P" (%)
Denmark	2.1	95.4	2.5
Germany	1.0	95.4	3.6
England	4.2	95.4	0.4
Netherlands	1.3	95.4	3.3
Scotland	2.9	95.4	1.7

Discussion

It must be kept in mind that the above analysis only gives an approximation to the truth, the most intricate thing being that P most certainly is not independent of age. It is nevertheless reasonable to conclude that for scale readings country differences exist, whereas this is not the case for otoliths. As to the measuring error, the data do not clearly indicate which sort of reading is to be preferred. The difference in mean ages for otoliths and scales is 0.0167 years, with a standard deviation of $\sqrt{2\sigma^2 : 30} \approx 0.0074$ and 40 degrees of freedom. This gives t = 2.26 with 5% > P > 2%, which indicates that scale and otolith readings should not be compared indiscriminately, and, for comparative purposes, only one method should be used.

4. Otolith type determination

The numbers of "wide" (W) and "narrow" (N) types are given in the following table:

Sample	Denmark		Germany		England		Netherlands		Scotland	
14 EA 61	31W	9N	28W	12N	27W	13N	-		24W	16N
18 EA 61	35W	3N	30W	8N	29W	9N	-		20W	18N
H 43	21W	19N	-		2W	38N	-		4W	36N
H 44	28W	15N	19W	24N	8W	35N	-		11W	32N
FR 22 Jul 1958	13W	22N	4W	31N	4W	31N	2W	33N	2W	33N
FR 16 Aug 1958	10W	29N	5W	34N	6W	33N	5W	34N	3W	36N

For the EA samples no Dutch data were available, and only fish that had been "typed" by all the other countries are used. For H 43 only the fish typed by Denmark, England and Scotland are used. For the FR samples the fish typed by all countries are used. The table clearly shows that the typing is not done in the same way in the different countries. Consequently, a statistical treatment of the data was not undertaken but the following table illustrates the discrepancies.

The figures are the numbers of otoliths which have been typed as indicated under Denmark, Germany, England, Netherlands and Scotland, e.g. line 9 means that in sample 14 EA 61 3 otoliths typed as "W" by Denmark have been typed as "N" by Germany, England and Scotland, whereas the figure was 5 and 9 for 18 EA 61 and H 44 respectively.

Denmark	Germany	England	Scotland	14 EA 61	18 EA 61	H 44
N	N	N	N	8	2	15
N	N	N	W			
N	N	W	N			
N	N	W	W			
N	W	N	N		1	
N	W	N	W			
N	W	W	N			
N	W	W	W	1		
W	N	N	N	3	5	9
W	N	N	W			
W	N	W	N	1	1	
W	N	W	W			
W	W	N	N	1	1	8
W	W	N	W	1		3
W	W	W	N	3	8	
W	W	W	W	22	20	8

Denmark	England	Scotland	H 43
N	N	N	19
N	N	W	
N	W	N	
N	W	W	
W	N	N	17
W	N	W	2
W	W	N	
W	W	W	2

Denmark	Germany	England	Netherlands	Scotland	FR 22 Jul 1958	FR 16 Aug 1958
N	N	N	N	N	22	29
N	N	N	N	W		
N	N	N	W	N		
N	N	N	W	W		
N	N	W	N	N		
N	N	W	N	W		
N	N	W	W	N		
N	N	W	W	W		
N	W	N	N	N		
N	W	N	N	W		
N	W	N	W	N		
N	W	N	W	W		
N	W	W	N	N		
N	W	W	N	W		
N	W	W	W	N		
N	W	W	W	W		
W	N	N	N	N	8	3
W	N	N	N	W		
W	N	N	W	N		1
W	N	N	W	W		
W	N	W	N	N		1
W	N	W	N	W		
W	N	W	W	N	1	
W	N	W	W	W		
W	W	N	N	N	1	
W	W	N	N	W		
W	W	N	W	N		
W	W	N	W	W		
W	W	W	N	N	1	1
W	W	W	N	W	1	
W	W	W	W	N		1
W	W	W	W	W	1	3

Appendix 1, Table 1 l_1 measurements (mm)

Sample	No.	Denmark	Germany	England	Netherlands	Scotland	Sum	Sum minus Denmark
14 E A 61	1	160	162	157	164	162	805	645
	2	105	115	105	112	110	547	442
	4	85	90	87	90	89	441	356
	6	135	135	137	144	137	688	553
	7	110	108	108	116	109	551	441
	10	140	145	144	150	148	727	587
	11	125	128	122	124	120	619	494
	18	110	105	108	118	111	552	442
	19	125	123	117	118	125	608	483
	26	95	95	98	102	99	489	394
	29	125	121	121	126	122	615	490
	32	120	128	121	126	123	618	498
	33	95	95	90	97	93	470	375
	34	160	170	165	167	167	829	669
	35	100	98	98	102	100	498	398
	36	95	97	93	99	92	476	381
	38	115	124	117	119	113	588	473
	39	120	128	123	128	124	623	503
	41	90	90	85	92	97	454	364
	43	110	108	111	113	110	552	442
	48	90	81	79	89	95	434	344
18 E A 61	1	95	94	92	99	95	475	380
	2	115	108	114	116	113	566	451
	6	95	100	91	98	96	480	385
	9	135	133	131	136	130	665	530
	11	150	148	150	153	151	752	602
	12	90	95	89	92	92	458	368
	15	145	158	150	148	150	751	606
	17	110	103	104	108	110	535	425
	19	110	114	110	113	113	560	450
	21	100	90	99	104	100	493	393
	22	90	100	91	98	94	473	383
	23	115	113	120	119	122	589	474
	24	85	81	82	85	87	420	335
	25	100	100	100	101	92	493	393
	27	95	94	90	93	95	467	372
	37	95	83	76	86	85	425	330
	39	120	125	109	126	117	597	477
	40	120	120	119	124	116	599	479
	47	120	117	116	118	119	590	470
Sum		4 495	4 522	4 419	4 613	4 523		18 077
Mean		112.38 (114.88)	113.05	110.48	115.32	113.08		112.98
H 43	12	90	96	94	99	96	475	385
	27	170	169	162	166	174	841	671
	29	130	130	128	134	130	652	522
	31	145	143	139	144	141	712	567
	39	135	138	135	139	141	688	553
	44	150	131	136	141	138	696	546
	46	150	144	145	148	148	735	585
	47	100	100	99	108	105	512	412
	48	95	98	92	97	96	478	383
	50	145	150	151	153	148	747	602
H 44	6	170	175	175	176	175	871	701
	10	145	152	145	151	149	742	597
	11	100	110	101	108	101	520	420
	17	140	146	145	145	145	721	581
	30	180	183	184	185	184	916	736
	35	135	138	139	141	138	691	556
	49	145	150	151	153	150	749	604
Sum		2 325	2 353	2 321	2 388	2 359		9 421
Mean		136.76 (139.26)	138.41	136.53	140.47	138.76		138.54

Appendix 1, Table 1 continued

Sample	No.	Denmark	Germany	England	Netherlands	Scotland	Sum	Sum minus Denmark
F R	2	150	147	150	153	150	750	600
22 Jul 1958	3	155	158	156	160	157	786	631
	5	140	140	143	150	149	722	582
	9	140	144	138	144	143	709	569
	11	150	143	138	160	142	733	583
	12	125	128	126	144	128	651	526
	13	145	146	142	138	144	715	570
	14	145	155	150	141	141	732	587
	15	160	158	159	161	161	799	639
	16	160	169	161	167	163	820	660
	17	110	111	112	113	115	561	451
	18	125	124	126	129	128	632	507
	20	135	130	133	135	134	667	532
	22	125	130	129	133	126	643	518
	23	135	145	139	143	144	706	571
	31	135	145	144	150	146	720	585
	35	145	143	144	146	146	724	579
	37	180	175	179	182	179	895	715
	39	175	170	167	173	169	854	679
	40	140	145	141	145	144	715	575
	41	155	159	161	156	159	790	635
	44	150	155	151	155	152	763	613
	46	120	126	124	124	122	616	496
	47	155	163	160	162	165	805	650
	49	150	151	151	154	154	760	610
	50	165	169	163	168	167	832	667
F R	1	145	149	149	147	146	736	591
16 Aug 1958	2	175	172	175	181	179	882	707
	5	175	185	181	187	185	913	738
	6	115	120	115	116	117	583	468
	12	115	112	114	118	117	576	461
	14	115	125	116	117	119	592	477
	15	100	94	98	100	99	491	391
	16	150	149	150	154	155	758	608
	19	105	105	101	106	104	521	416
	25	115	111	114	113	116	569	454
	26	125	123	128	134	132	642	517
	33	135	134	135	138	137	679	544
	37	100	117	98	103	101	519	419
	39	100	92	95	96	98	481	381
	41	145	147	147	148	152	739	594
	42	115	94	92	98	96	495	380
	43	105	104	104	110	106	529	424
	45	100	95	98	102	103	498	398
	46	110	104	102	105	104	525	415
Sum		6 115	6 161	6 099	6 259	6 194		24 713
Mean		135.89 (138.39)	136.91	135.53	139.09	137.64		137.29

Appendix 1, Table 2 l_2 measurements (mm)

Sample	No.	Denmark	Germany	England	Netherlands	Scotland	Sum	Sum minus Denmark
14 E A 61	1	225	220	221	227	220	1 113	888
	2	170	175	176	182	179	882	712
	4	180	182	184	185	188	919	739
	7	195	193	199	203	198	988	793
	10	210	212	212	219	214	1 067	857
	11	200	199	201	201	199	1 000	800
	18	180	189	187	195	195	946	766
	23	175	177	175	179	181	887	712
	26	180	197	198	202	180	957	777
	32	210	208	211	213	210	1 052	842
	33	190	197	197	202	194	980	790
	35	210	199	196	204	199	1 008	798
	36	175	180	179	184	179	897	722
	38	205	218	207	211	206	1 047	842
	39	185	206	208	200	210	1 009	824
	41	185	190	188	192	195	950	765
	43	185	188	187	191	188	939	754
	48	185	189	191	194	202	961	776
18 E A 61	1	175	176	178	182	177	888	713
	2	185	194	190	193	190	952	767
	6	195	200	191	196	194	976	781
	9	210	208	208	213	212	1 051	841
	11	215	212	216	219	216	1 078	863
	12	175	180	176	178	177	886	711
	15	225	228	229	238	223	1 143	918
	17	195	196	197	201	200	989	794
	19	185	183	184	192	188	932	747
	21	190	195	193	195	194	967	777
	22	190	193	190	196	192	961	771
	23	195	200	203	196	204	998	803
	24	180	175	177	179	179	890	710
	25	195	198	198	203	195	989	794
	27	180	178	179	181	182	900	720
	37	175	184	179	186	190	914	739
	39	195	200	190	203	197	985	790
	40	200	200	200	204	199	1 003	803
	47	200	197	191	200	193	981	781
Sum		7 105	7 216	7 186	7 339	7 239		28 980
Mean		192.03 (194.53)	195.03	194.22	198.35	195.65		195.81
H 43	12	185	190	189	193	192	949	764
	27	235	234	235	238	239	1 181	946
	29	200	202	200	202	204	1 008	808
	31	215	219	214	219	214	1 081	866
	39	215	218	212	215	217	1 077	862
	44	210	223	226	216	225	1 100	890
	46	235	235	236	239	234	1 179	944
	47	195	194	196	197	196	978	783
	48	190	190	190	192	190	952	762
	50	215	212	216	216	210	1 069	854
H 44	6	235	239	237	239	236	1 186	951
	10	240	242	244	247	244	1 217	977
	11	175	180	178	178	176	887	712
	17	225	213	227	229	228	1 122	897
	30	245	248	248	246	246	1 233	988
	49	215	213	217	220	215	1 080	865
Sum		3 430	3 452	3 465	3 486	3 466	17 299	13 869
Mean		214.38 (216.88)	215.75	216.56	217.88	216.62	216.24	216.70

Appendix 1, Table 2 continued

Sample	No.	Denmark	Germany	England	Netherlands	Scotland	Sum	Sum minus Denmark
F R	2	230	232	232	234	234	1 162	932
22 Jul 1958	3	235	241	241	239	239	1 195	960
	9	220	218	221	221	221	1 101	881
	12	205	207	213	219	212	1 056	851
	13	230	230	225	229	228	1 142	912
	14	225	228	227	228	229	1 137	912
	15	220	220	219	223	221	1 103	883
	16	240	245	242	245	246	1 218	978
	17	185	179	189	188	186	927	742
	18	230	228	229	228	229	1 144	914
	20	230	230	231	232	231	1 154	924
	22	205	207	209	209	209	1 039	834
	23	220	221	222	223	223	1 109	889
	31	210	221	216	219	216	1 082	872
	35	225	223	223	227	226	1 124	899
	37	235	231	238	239	239	1 182	947
	39	225	227	229	231	230	1 142	917
	40	220	225	222	221	226	1 114	894
	41	230	235	241	242	238	1 186	956
	44	220	227	227	229	229	1 132	912
	46	205	210	209	210	214	1 048	843
	47	225	230	229	231	231	1 146	921
	49	220	225	223	225	224	1 117	897
F R	1	215	215	218	216	219	1 083	868
16 Aug 1958	2	225	224	224	228	227	1 128	903
	3	190	195	190	197	192	964	774
	5	230	235	237	239	239	1 180	950
	6	185	187	182	186	189	929	744
	8	200	197	195	202	201	995	795
	12	210	208	213	212	215	1 058	848
	15	190	196	195	196	195	972	782
	16	220	220	227	225	224	1 116	896
	19	190	190	188	192	191	951	761
	25	190	187	188	193	194	952	762
	26	200	198	202	204	206	1 010	810
	33	210	211	207	212	212	1 052	842
	35	180	178	182	184	184	908	728
	39	180	178	179	185	184	906	726
	41	230	239	236	242	241	1 188	958
	42	170	178	175	182	178	883	713
	43	195	195	193	205	196	984	789
	45	190	181	190	187	187	935	745
	46	190	194	195	196	195	970	780
Sum		9 080	9 146	9 173	9 275	9 250		36 844
Mean		211.16 (213.66)	212.70	213.33	215.70	215.12		214.21

Appendix 1, Table 3 Age determinations

Sample	No.	Denmark	Germany	England	Netherlands	Scotland	Sum
	1.	Scales					
14 E A 61	47	2 745	2 809	2 745	2 809	2 745	13 853
18 E A 16	44	2 955	2 955	2 909	2 932	2 886	14 637
H 43	42	3 929	4 024	3 833	3 952	3 929	19 667
H 44	46	3 500	3 478	3 413	3 500	3 478	17 369
F R 22 Jul 1958	45	3 200	3 178	3 156	3 222	3 222	15 978
F R 16 Aug 1958	49	2 102	2 122	2 122	2 122	2 082	10 550
Sum		18 431	18 566	18 178	18 537	18 342	92 054
Mean		3 072	3 094	3 030	3 090	3 057	3 068
	2.	Otoliths					
14 E A 61	47	2 766	2 766	2 745	2 745	2 766	13 788
18 E A 61	44	2 955	2 955	2 955	2 955	2 932	14 752
H 43	42	3 952	3 952	3 905	4 000	3 976	19 785
H 44	46	3 500	3 522	3 522	3 478	3 478	17 500
F R 22 Jul 1958	45	3 244	3 289	3 200	3 289	3 200	16 222
F R 16 Aug 1958	49	2 102	2 102	2 102	2 102	2 102	10 510
Sum		18 519	18 586	18 429	18 569	18 454	92 557
Mean		3 086	3 098	3 072	3 095	3 076	3 085

Appendix 2 HERRING RECRUITS RACIAL SURVEYS - SUMMARY OF COMPUTER PROGRAMS

(The four surveys concerned took place in the spring and autumn of 1960 and 1961)

Program reference no.	Approximate dates of processing	Form of data	Form of results	Reference no. of results sheet	Remarks
B 252 Version G	Sep 1964-May 1965	DENARY. A number of samples, each consisting of: (a) Day, month and year of sample; sample reference number; rectangle location reference (a letter and a number). (b) Details of a number of fish, each fish having 11 characters, namely - fish reference number; l_1; total length; sex (F or M); maturity stage; VS; K_2; year-class; otolith type (narrow or wide); pectoral fin rays; gillrakers. In the large number of cases where characters were missing from the original data, Q (query) or Z (not examined) was punched. Each sample was terminated with the letter B, except the last of each survey which ended with the letter C.	DENARY. Regressions of l_1 on total length for maturity stages 1 and 2 separately and combined, each region (W, NE and SE) separately and combined, each of the 3 year-classes 1957, 1958, 1959, each survey. BINARY. Visible punch "B252". A tape for each survey includes all fish (but not all characters). Each sample is represented by 3 binary blocks as follows: (a) 9 stores containing - number of fish in the sample (say, n); date (3 numbers); sample number; numerical value of rectangle letter; rectangle number; regional indices (2). (b) n stores, each containing fish characters packed thus: 38 | 28 | 21 | 14 | 7 Fish no. | Year-class | Maturity stage | VS | K_2 (c) 2n stores containing alternatively l_1 and total length in cm (floating point). The end of the tape for each survey is signified by a block similar to (a) above, having the first store zero. Q or Z appears as 100 or 99 respectively.	-	Due to a variety of causes, but mainly on account of the large number of lettershifts and letters in the data, great difficulty was experienced in getting the original data tapes correct. Denary results were used for calculation of standard l_1 in subsequent processing. Binary results were used as data for subsequent processing (much faster read routine on computer). See B254, 254B, 276.
B 254	Mar-May 1965	DENARY. Details of the particular survey and year-class required.	DENARY. For a single year-class (determined by denary output): (i) Summary by region of the numbers of fish with and without VS and K_2 within credible range (maturity stages I and II only). (ii) Number of fish, mean and variance, for each rectangle and each region, for the characters: (a) total length (b) l_1 (c) VS - 50 (d) K_2 - 10	1-4	

<table>
<tr><th>Program reference no.</th><th>Approximate dates of processing</th><th>Form of data</th><th>Form of results</th><th>Reference no. of results sheet</th><th>Remarks</th></tr>
<tr><td>B 254</td><td>Mar–
May 1965</td><td><u>BINARY</u>. Results tape from B252.</td><td><u>BINARY</u>. For the same single year-class, visible "B 254" followed by identifying indication, e.g. "S 60 Y 57" for spring 1960 survey, 1957 year-class.

Each sample containing fish of maturity stages I and II, and of the required year-class, is represented by 3 blocks:
(a) 7 stores holding - number of fish of maturity stages I and II, of the particular year-class, in the sample; date (3 numbers); sample number; rectangle reference (1 number); regional reference (1 number).
(b) Details of the selected fish, packed as in the similar block of B252.
(c) Further details of the same fish, packed thus:

38 | 21 | 7
Length (mm) | l_1 (mm) | Indicator

the indicator being 0, 3, 6 or 9 to show:
VS and K_2 both absent - 0
VS absent, K_2 present - 3
VS present, K_2 absent - 6
VS and K_2 both present - 9</td><td></td><td><u>Binary results</u>. Used for subsequent processing.
See B255, 258, 259.
Also used for producing experimental tape for Principal Components Analysis.
See B257.</td></tr>
<tr><td>B 255
Version D</td><td>May 1965</td><td><u>DENARY</u>. Defining the required survey and year-class, coefficients for regression of l_1 on total length, coefficients for discriminant functions, minima and maxima of discriminant values.</td><td><u>DENARY</u>. (i) Number of fish, mean and variance, for each rectangle and each region, for the following:
(a) Calculated l_1 (standard)
(b) Discriminant values X_1 } using l_1 and
(c) " " Y_1 } VS only
(d) " " X_2 } using l_1, VS
(e) " " Y_2 } and K_2</td><td>5-9</td><td></td></tr>
<tr><td></td><td></td><td><u>BINARY</u>. Results tape from B254.</td><td>(ii) Plot of X_1, Y_1 distribution (in $\frac{1}{2}$ unit groups) for each of the 3 regions separately, and numbers allocated to each stock.

(iii) Plot of X_2, Y_2 distribution and allocation to stock, for each region.</td><td>10 A, B, C

11 A, B, C</td><td>Some data was processed by versions of the program earlier than D which did not include allocation to stock, but see B258.</td></tr>
</table>

Program reference no.	Approximate dates of processing	Form of data	Form of results	Reference no. of results sheet	Remarks
B 258	Jun 1965	DENARY and BINARY As for B255.	DENARY. Number of fish allocated to each stock, for each of the 3 regions, using (a) l_1 and VS only, and (b) l_1, VS and K_2.	(10 and 11)	Supplementary to B255 for versions earlier than D.
B 259	Jun 1965	DENARY and BINARY As for B255 but using different discriminant coefficients.	DENARY. As for B255, but omitting means etc. of standard l_1.	12-17	Version X of the programme omits plots of X, Y distribution (used for S61 Y57).
B 254 Version B	Nov 1965	DENARY and BINARY As for original version.	DENARY. For all maturity stages, a table of means of fish characters, by maturity stage, by region. BINARY. As for original version but including all maturity stages.	18	Used for S60 Y57 and S61 Y58 only.
B 255 Version E	Nov 1965	DENARY. As for Version D. BINARY. Results tape from B254 B.	DENARY. As for Version D of B255, but these results include all maturity stages.	19-25	Used for S60 Y57 and S61 Y58 only.
B 275	Jun-Jul 1966	No data.	Trials of bivariate probability integral.	-	Preparation for B277.
B 276	Sep/Oct 1966 and Mar 1967	DENARY. Details of particular survey of year-class required. BINARY. Results tape from B252.	DENARY. For all maturity stages: (i) Summary by region of the numbers of fish with and without VS and K_2 within credible range. (ii) Number of fish, mean and variance, for each rectangle and each region, for (a) total length and (b) standard l_1. BINARY. As for B254, but including fish of all maturity stages.	26 (total length)	The "charts" for standard l_1 were incorrect owing to error in program Binary results used for subsequent processing. See B277, 254 D, 259 Y, 277 X, 259 H, 269.
B 277 Versions up to G	Sep 1966-Mar 1967	DENARY. Defining survey and year-class; initial mean stock characters and variances; characters to be used for discrimination; area of the North Sea to be included.	DENARY. Table giving stock centres, stocks as classified, adjusted numbers in each stock, Δl_1 and adjusted l_1 variance, for each iteration. Versions up to G. Program testing, mainly with an artificial sample.		Most runs used characters l_1, VS and K_2. Runs using l_1 and VS only were done for the 1958 year-class, all surveys (results sheet 39).
Version H	Apr-May 1967	BINARY. Results tape from B276.	Version H. Live runs with original areas.	31-35 and 39	
Versions K-M	May-Sep 1967		Versions K-M. Revised areas introduced.	36-38	

Appendix 2 continued

Program reference no.	Approximate dates of processing	Form of data	Form of results	Reference no. of results sheet	Remarks
B 254 Version D	Jun 1967	DENARY. Details of survey and year-class required. BINARY. Results tape from B276.	DENARY. As for original version but these results include fish of all maturity stages. BINARY. Nil.	27-30	A60 Y57, S61 Y57 and 59, A61 Y58 only processed. (In other cases, not much difference between numbers for maturity stages I and II and for all maturity stages.)
B 259 Version Y	Jun 1967	DENARY. Defining the required survey and year-class; coefficients for regression of l_1 on total length; coefficients for discriminant functions; and stock classification limits. BINARY. Results tape from B276.	DENARY. "Charts" of number of fish, mean and variance, for each rectangle and each region, for the discriminant values X and Y using: (1) l_1 and VS only (2) l_1, VS and K_2 (3) VS and K_2 only; and numbers allocated to each stock by region.	41-46	The discriminant coefficients used here were derived from mean stock characters and variances as adjusted by the average "Δl_1" for the year-class (see sheet 40).
B 277 Version XA Version XB	Jul 1967 Sep 1967	BINARY. Results tape from B276.	DENARY. The stock numbers as classified in B259 Y above, and as adjusted to compensate for the probability of misclassification.	47	
B 259 Version H	Aug 1967	DENARY and BINARY As for B259 Y.	DENARY. Histograms of discriminant values X and Y.	-	Used for S60 Y57 only.
B 269 Version A	Sep-Oct 1967	DENARY. Giving all relevant coefficients, limits, etc. BINARY. Results tape from B276.	DENARY. Numbers allocated to each stock and adjusted for misclassification, by revised areas.	48	
B 269 Version B	Jan 1968	As for version A, but denary tape also defines areas to be selected.	DENARY. Classified and adjusted numbers for each selected (high density) area.	-	
B 276A B 277N B 277Z	Feb-Mar 1968	Similar to other versions.	For 1960 year-class, similar to the results for 1957, 1958 and 1959 year-classes dealt with previously.		

Appendix 3 FISHING POWER AND ABUNDANCE ESTIMATES

by K. P. Andersen, Danmarks Fiskeri- og Havundersøgelser, Charlottenlund

The absolute density of herring (number per area unit) in the North Sea is taken as a function of locality and time:

density = $D = D(\beta, \lambda, t)$ herring per unit of area,

where β and λ are latitude and longitude, and t is time. For the total number of herring in the North Sea we have:

$$\text{Total number} = N_{\text{North Sea}} = \int_{\text{North Sea}} D \, d\,\omega \quad .$$

D is determined by fishing and as an estimate of D we use the catch per hour, c/f. The best we can hope for is that c/f is proportional to D or

$$c/f = p(\text{ship},\ t).\ D \tag{1}$$

where p is a function depending on the ship and time only. p may be considered as the fishing power and it is assumed to be time dependent.

To be able to utilize the data from the young herring cruises we have to specify assumptions about p and D and to simplify by using approximations. The first assumption we shall make is that p and D change slowly as t changes. This means that we can write

$$c/f \approx p(\text{ship},\ t_i).\ D(\beta,\ \lambda,\ t_i)$$

where t_i is the mean date of the i'th cruise.

D is a very complicated function of λ and β and there is no hope of approximating to it by a simple surface. In addition we cannot expect equation (1) to hold exactly, because c/f is subjected to random variation, and instead of (1) we should write

$$c/f = p.D + \Sigma \tag{2}$$

where Σ is a random component with zero mean.

This means that we are not able to obtain a detailed knowledge of D even if we can take t as constant in each cruise. If we think of a domain B we have:

$$\text{Abundance} = \text{number of fish in B} = A_B = \int_B^{D} (\beta,\ \lambda,\ t)\ d\,\omega .$$

If ϕ_B (x) is the area of the part of B where $D \leq x$, and we make D a random variable by choosing random positions in B, then:

The probability for $D \leq x = P(D \leq x) = \frac{1}{B} \varphi_B (x)$.

This means that if a station S is chosen at random inside B then the density x is a random variable with density distribution

$$\frac{1}{\text{Area of B}} \cdot \frac{d\varphi}{dx} = \frac{1}{\alpha(B)} \cdot \frac{d\varphi}{dx}.$$

For the mean and variance we find:

$$\text{Mean } x = E(x) = \int_0^\infty x. \frac{1}{\alpha(B)} \cdot \frac{d\varphi}{dx} \, dx = \frac{1}{\alpha(B)} \int_{\text{North Sea}} x. d\varphi = \frac{A_B}{\text{Area of B}}$$

$$\text{Var } x = \int_0^\infty \left(x - \frac{A_B}{\alpha(B)}\right)^2 \cdot \frac{1}{\alpha(B)} \cdot \frac{d\varphi}{dx} \, dx.$$

For c/f we have

$$c/f = p. x + \Sigma,$$

and supposing Σ to be independent of x, c/f is a random variable with

$$E(c/f) = p. E(x) + E\Sigma = p. \frac{A_B}{\alpha(B)}$$

$$\text{Var } c/f = p^2. \text{Var } x + \text{Var } \Sigma.$$

Σ is in general not independent of x, since a greater x will give a greater Var Σ, but the more uniform x is in B the more reasonable is it to take Σ as being independent of x.

Var Σ is assumed to be independent of f. The reason for this is the shoaling habits of herring. We are actually saying that a two-hour haul can be regarded as the sum of two completely correlated one-hour hauls and not two uncorrelated one-hour hauls.

If we take two areas B and C where $\frac{A_C}{\alpha(C)} = k \frac{A_B}{\alpha(B)}$ we shall suppose that $\frac{1}{k} x_c$ is distributed as x_B. By doing this we assume a certain similarity in dense and sparse herring areas. This assumption implies that:

$$\text{Var } x_c = \int_0^\infty \left(y - \frac{A_C}{\alpha(C)}\right)^2 \frac{1}{\alpha(C)} \frac{d\phi_C(y)}{dy}\, dy$$

$$= \int_0^\infty \left(y - \frac{A_C}{\alpha(C)}\right)^2 \frac{1}{\alpha(B)} \frac{d\phi_B(\frac{y}{k})}{dy}\, dy$$

$$= \int_0^\infty \left(ku - \frac{k\, A_B}{\alpha(B)}\right)^2 \frac{1}{\alpha(B)} \frac{d\phi_B(u)}{k\,du}\, k\,du = k^2 \text{ Var } x_B ,$$

or

$$\frac{\text{Var } x_c}{E^2(x_c)} = \frac{k^2 \text{ Var } x_B}{k^2\, E(x_B)} = \frac{\text{Var } x_B}{E^2(x_B)} = C_1 .$$

If $V(\Sigma) = C_2 .\, p^2 .\, E^2(x)$ (where C_2 is a constant), an assumption that expresses similarity between ships and areas in the random component, we find

$$\frac{\text{Var } c/e}{\overline{cpr}^2} = \frac{V(\frac{c}{f})}{E^2(\frac{c}{f})} = \frac{p^2 . \text{Var } x + \text{Var } \Sigma}{p^2 . E^2(x)} = \frac{\text{Var } x}{E^2(x)} = \frac{C_2 . p^2 . E^2(x)}{p^2 . E^2(x)} = C_1 + C_2$$

independent of both area and ship.

To obtain uniformity in x one has to split up the North Sea into smaller areas, and the smaller the better. To eliminate the effect of Σ one must have many hauls in each area. The two demands are working against each other, so we have to compromise. In Figure B I 13 is shown a division into five areas. The standard statistical rectangles have been grouped into areas in which the densities are sufficiently uniform and contain a high number of hauls. It is an attempt to take areas of biological uniformity. We can make a rough check of our assumptions by calculating the ratio between the standard deviation and the corresponding mean. In App. 3, Figure 1 is plotted the ratio of the standard deviation of x on $\bar{x}$ against $\bar{x}$, and the ratio is seen to be reasonably randomly distributed about the mean ratio 2. 32. In this way we have obtained an estimate of:

$$\sqrt{C_1 + C_2} \approx 2.32 .$$

We can now state a mathematical model for the single hauls:

$$x_{ijkl} = p_{ij} \cdot D_{kj} + \Delta_{ijkl} ,$$

where x_{ijkl} is the l'th catch per hour in the k'th area taken by the i'th ship in the j'th season. p_{ij} is the fishing power of the i'th ship in the j'th season, D_{kj} the mean density of herring in the k'th area in the j'th season, and the Δ's are random components. The Δ's are assumed to be independent and to have zero means and variances proportional to $(p_{ij} \cdot D_{kj})^2$.

For the mean c/f we find

$$\bar{x}_{ijk} = p_{ij} \cdot D_{kj} + \frac{\Sigma_l \cdot \Delta_{ijkl}}{n_{ijk}} = p_{ij} \cdot D_{kj} + \Sigma_{ijk} , \qquad (3)$$

where n is the number of hauls and Σ is a random variable, approximately normally distributed, with zero mean and variance proportional to $(p_{ij} \cdot D_{kj})^2/n_{ijk}$. If now the p's and D's were approximately known, say:

$$p_{ij} \approx \hat{p}_{ij} + \Delta p_{ij}$$

$$D_{kj} \approx \hat{D}_{kj} + \Delta D_{kj}$$

where the $\hat{p}$'s and $\hat{D}$'s are known and the Δp's and ΔD's are known to be small from (3) we can write

$$\frac{\bar{x}_{ijk}}{\hat{p}_{ij} \cdot \hat{D}_{kj}} = (1 + \frac{\Delta p_{ij}}{\hat{p}_{ij}}) \; (1 + \frac{\Delta D_{kj}}{\hat{D}_{kj}}) + \frac{\Sigma_{ijk}}{\hat{p}_{ij} \cdot \hat{D}_{kj}}$$

$$= 1 + \frac{\Delta p_{ij}}{\hat{p}_{ij}} + \frac{\Delta D_{kj}}{\hat{D}_{kj}} + \frac{(\Delta p_{ij})\,(\Delta D_{kj})}{\hat{p}_{ij} \cdot \hat{D}_{kj}} + \frac{\Sigma_{ijk}}{\hat{p}_{ij} \cdot \hat{D}_{kj}}$$

$$\approx 1 + \frac{\Delta p_{ij}}{\hat{p}_{ij}} + \frac{\Delta D_{kj}}{\hat{D}_{kj}} + \eta_{ijk} \qquad (4)$$

where the η's are independent and approximately normally distributed, with zero mean and variance proportional to $1/\eta_{ijk}$. The factor of proportionality is $(C_1 + C_2)$. From (4) it is seen that

$$\frac{\Delta p}{\hat{p}} \text{ and } \frac{\Delta D}{\hat{D}}$$

as well as $(C_1 + C_2)$ can be estimated by a two-way classification.

The two-way classification gives us estimates of the differences between single ships and the mean of all ships in each survey.

$$\frac{\Delta p}{\hat{p}} - \text{mean } \frac{\Delta p}{\hat{p}} = \frac{\Delta p}{\hat{p}} - m_p \qquad \left(\text{say } \frac{\widetilde{\Delta p}}{\hat{p}}\right)$$

$$\frac{\Delta D}{\hat{D}} - \text{mean } \frac{\Delta D}{\hat{D}} = \frac{\Delta D}{\hat{D}} - m_D \qquad \left(\text{say } \frac{\widetilde{\Delta D}}{\hat{D}}\right)$$

$$\mu = 1 + m_p + m_D \qquad (\text{say } \tilde{\mu})$$

Mean catches are estimated by

$$\hat{p}\hat{D}\left(\tilde{\mu} + \frac{\widetilde{\Delta p}}{\hat{p}} + \frac{\widetilde{\Delta D}}{\hat{D}}\right).$$

The estimates

$$\tilde{p} = \hat{p}\left(1 + \frac{\widetilde{\Delta p}}{p}\right)$$

$$\tilde{D} = \hat{D}\left(1 + \frac{\widetilde{\Delta D}}{D}\right.$$

are only relative estimates, since

$$E(\tilde{p}) = \hat{p}\left(1 + \frac{\Delta p}{\hat{p}} - m_p\right) \approx \hat{p}\left(1 + \frac{\Delta p}{\hat{p}}\right)(1 - m_p) = p(1 - m_p)$$

$$E(\tilde{D}) = \hat{D}\left(1 + \frac{\Delta D}{\hat{D}} - m_D\right) \approx \hat{D}\left(1 + \frac{\Delta D}{\hat{D}}\right)(1 - m_D) = D(1 - m_D)$$

Actually we have no approximate p's and D's and instead we have determined for each season the $\tilde{p}$'s and $\tilde{D}$'s that give all Δp's and ΔD's equal to zero. The values found in this way are almost unbiased relative estimates of the p's and D's if Var η is small as compared with 1. The areas chosen are such that the numbers of hauls are large, and as η is proportional to 1/number of hauls this term is generally small. The mean c/f's, together with the results of the analysis, are shown in App. 3, Table 1. The standard deviations of the estimates of density and fishing power are rather large, which makes it impossible to quantify the absolute differences between ships and areas. However, real differences between them exist.

Appendix 3, Table 1 Estimates of fishing power and density

Spring 1960

Area	1 "Ernest Holt"	2 "Scotia"	3 "Anton Dohrn"	4 "Willem Beukelsz"	Density $\tilde{D}_l$	Standard deviation (%)
	Mean catch/hour by ship and area (numbers of fish) (Number of hauls in parentheses)					
1		99.71 (42)	681.5 (70)		44.30	47
2			532.2 (25)		34.59	55
3	537.3 (18)			632.6 (27)	2 262	48
4	26.18 (22)		6 348 (19)	638.6 (6)	412.6	34
5	97.00 (4)			480.6 (13)	870.7	61
Fishing power, $\tilde{p}_l$	0.1396	2.251	15.38	0.5200		
Standard deviation (%)	41	49	37	50		

Area	1 "Ernest Holt"	2 "Scotia"	3 "Anton Dohrn"	4 "Willem Beukelsz"
	Calculated mean catch/hour by ship and area (numbers of fish)			
1		99.71	681.5	
2			532.2	
3	315.8			1 176
4	57.59		6 348	214.6
5	121.5			452.8

$\{\tilde{p}_{il} \cdot \tilde{D}_{jl}\}$

Variances and covariances

(1) Fishing power; $\mathrm{Var}\,\frac{p_{il}}{p_{il}} = (C_1+C_2)\,\lambda_{ii}$; $\mathrm{Cov}\,\left(\frac{\tilde{p}_{il}}{p_{il}}, \frac{\tilde{p}_{kl}}{p_{kl}}\right) = (C_1+C_2)\,\lambda_{ik}$

$\{\lambda_{ik}\}$

	1 "Ernest Holt"	2 "Scotia"	3 "Anton Dohrn"	4 "Willem Beukelsz"
1. "Ernest Holt"	0.0313	-0.0265	-0.0170	0.0121
2. "Scotia"	-0.0265	0.0446	0.0160	-0.0342
3. "Anton Dohrn"	-0.0170	0.0160	0.0256	-0.0247
4. "Willem Beukelsz"	0.0121	-0.0342	-0.0247	0.0467

(2) Density; $\mathrm{Var}\,\frac{D_{jl}}{D_{jl}} = (C_1+C_2)\,\mu_{jj}$; $\mathrm{Cov}\,\left(\frac{\tilde{D}_{jl}}{D_{jl}}, \frac{\tilde{D}_{kl}}{D_{kl}}\right) = (C_1+C_2)\,\mu_{jk}$ $(j \neq k)$

$\{\mu_{jk}\}$

Area	Area 1	Area 2	Area 3	Area 4	Area 5
1	0.0405	0.0211	-0.0251	-0.0025	-0.0341
2	0.0211	0.0560	-0.0302	-0.0076	-0.0392
3	-0.0251	-0.0302	0.0421	-0.0012	0.0144
4	-0.0025	-0.0076	-0.0012	0.0214	-0.0102
5	-0.0341	-0.0392	0.0144	-0.0102	0.0691

Appendix 3, Table 1 continued

Autumn 1960

Area	1 "Sir Lancelot"	2 "Scotia"	3 "Anton Dohrn"	4 "Willem Beukelsz"	5 "Dana"	Density $\tilde{D}_l$	Standard deviation (%)
	Mean catch/hour by ship and area (numbers of fish) (Number of hauls in parentheses)						
1	60.24 (17)	14.03 (38)			0.5000 (4)	101.8	54
2					3.143 (21)	4.601	50
3	336.7 (13)		655.9 (69)	462.1 (29)	6.000 (1)	744.8	29
4	7.333 (3)		1401 (54)	169.8 (4)	1305 (50)	1715	27
5			0.2222 (9)	2.214 (14)	1.000 (4)	1.862	44
Fishing power, $\tilde{p}_l$	0.4819	0.1378	0.8032	0.7431	0.6829		
Standard deviation (%)	38	56	30	39	36		

$\{\tilde{p}_{il} \cdot \tilde{D}_{jl}\}$

Area	1 "Sir Lancelot	2 "Scotia"	3 "Anton Dohrn"	4 "Willem Beukelsz"	5 "Dana"
	Calculated mean catch/hour by ship and area (numbers of fish)				
1	49.06	14.03			69.52
2					3.142
3	358.9		598.2	553.5	508.6
4	826.5		1377	1274	1171
5			1.496	1.384	1.272

Variances and covariances

(1) Fishing power; $\mathrm{Var}\,\frac{p_{il}}{p_{il}} = (C_1+C_2)\,\lambda_{ii}$; $\mathrm{Cov}\,(\frac{\tilde{p}_{il}}{p_{il}}, \frac{\tilde{p}_{kl}}{p_{kl}}) = (C_1+C_2)\,\lambda_{ik}$

$\{\lambda_{ik}\}$

	1 "Sir Lancelot"	2 "Scotia"	3 "Anton Dohrn"	4 "Willem Beukelsz"	5 "Dana"
1. "Sir Lancelot"	0.0264	0.0045	-0.0086	-0.0113	-0.0110
2. "Scotia"	0.0045	0.0592	-0.0207	-0.0240	-0.0190
3. "Anton Dohrn"	-0.0086	-0.0207	0.0171	0.0066	0.0056
4. "Willem Beukelsz"	-0.0113	-0.0240	0.0066	0.0287	0.0000
5. "Dana"	-0.0110	-0.0190	0.0056	0.0000	0.0244

(2) Density; $\mathrm{Var}\,\frac{D_{jl}}{D_{jl}} = (C_1+C_2)\,\mu_{jj}$; $\mathrm{Cov}\,(\frac{\tilde{D}_{jl}}{D_{jl}}, \frac{\tilde{D}_{kl}}{D_{kl}}) = (C_1+C_2)\,\mu_{jk}$ $(j \neq k)$

$\{\mu_{jk}\}$

Area	Area 1	Area 2	Area 3	Area 4	Area 5
1	0.0563	-0.0196	-0.0079	-0.0109	-0.0180
2	-0.0196	0.0473	-0.0110	-0.0022	-0.0145
3	-0.0079	-0.0110	0.0158	0.0024	0.0007
4	-0.0109	-0.0022	0.0024	0.0137	-0.0032
5	-0.0180	-0.0145	0.0007	-0.0032	0.0349

Appendix 3, Table 1 continued

Spring 1961

Area	1 "Scotia"	2 "Anton Dohrn"	3 "Willem Beukelsz"	4 "Dana"	Density $\tilde{D}_1$	Standard deviation (%)
	Mean catch/hour by ship and area (numbers of fish) (Number of hauls in parentheses)					
1	90.96 (52)	76.01 (74)			79.12	47
2		391.0 (29)		459.7 (12)	416.4	41
3		765.5 (4)	371.3 (30)	2.000 (5)	419.5	41
4		12.33 (3)	306.2 (13)	611.0 (40)	501.6	36
5			499.0 (22)	255.5 (4)	537.0	49
Fishing power, $\tilde{p}_1$	1.1496	0.9606	0.8450	1.0480		
Standard deviation (%)	44	32	46	37		

Area	1 "Scotia"	2 "Anton Dohrn"	3 "Willem Beukelsz"	4 "Dana"
	Calculated mean catch/hour by ship and area (numbers of fish)			
1	90.96	76.00		
2		400.0		435.0
3		403.0	345.5	439.6
4		481.8	423.9	525.7
5			453.8	562.8

$\left\{ \tilde{p}_{il} \cdot \tilde{D}_{jl} \right\}$

Variances and covariances

(1) Fishing power; $\mathrm{Var}\,\dfrac{p_{il}}{p_{il}} = (C_1+C_2)\,\lambda_{ii}$; $\mathrm{Cov}\left(\dfrac{\tilde{p}_{il}}{p_{il}}, \dfrac{\tilde{p}_{kl}}{p_{kl}}\right) = (C_1+C_2)\,\lambda_{ik}$

$\left\{\lambda_{ik}\right\}$

	1 "Scotia"	2 "Anton Dohrn"	3 "Willem Beukelsz"	4 "Dana"
1. "Scotia"	0.0360	0.0114	-0.0269	-0.0205
2. "Anton Dohrn"	0.0114	0.0196	-0.0187	-0.0123
3. "Willem Beukelsz"	-0.0269	-0.0187	0.0389	0.0067
4. "Dana"	-0.0205	-0.0123	0.0067	0.0261

(2) Density; $\mathrm{Var}\,\dfrac{D_{jl}}{D_{jl}} = (C_1+C_2)\,\mu_{jj}$; $\mathrm{Cov}\left(\dfrac{\tilde{D}_{jl}}{D_{jl}}, \dfrac{\tilde{D}_{kl}}{D_{kl}}\right) = (C_1+C_2)\,\mu_{jk}$ $(j \neq k)$

$\left\{\mu_{jk}\right\}$

Area	Area 1	Area 2	Area 3	Area 4	Area 5
1	0.0416	0.0144	-0.0180	-0.0125	-0.0254
2	0.0144	0.0311	-0.0162	-0.0076	-0.0218
3	-0.0180	-0.0162	0.0308	-0.0019	0.0053
4	-0.0125	-0.0076	-0.0019	0.0248	-0.0028
5	-0.0254	-0.0218	0.0053	-0.0028	0.0447

Appendix 3, Table 1 continued

Autumn 1961

Area	1 "Scotia"	2 "Anton Dohrn"	3 "Willem Beukelsz"	4 "Dana"	Density $\tilde{D}_1$	Standard deviation (%)
	Mean catch/hour by ship and area (numbers of fish) (Number of hauls in parentheses)					
1	82.74 (27)	0.0000 (1)		5.800 (10)	24.28	59
2				3519 (11)	11416	72
3	5.333 (3)	1579 (63)	541.9 (11)		695.3	39
4		2456 (49)		423.0 (14)	1188	39
5			44.20 (20)		56.71	78
Fishing power, $\tilde{p}_1$	3.0556	2.1628	0.7794	0.3082		
Standard deviation (%)	58	38	64	45		

Area	1 "Scotia"	2 "Anton Dohrn"	3 "Willem Beukelsz"	4 "Dana"
	Calculated mean catch/hour by ship and area (numbers of fish)			
1	74.19	52.51		7.483
2				3518
3	2125	1504	541.9	
4		2569		366.1
5			44.20	

$\left\{ \tilde{p}_{il} \cdot \tilde{D}_{jl} \right\}$

Variances and covariances

(1) Fishing power; $\mathrm{Var}\,\frac{p_{il}}{p_{il}} = (C_1+C_2)\,\lambda_{il}$; $\mathrm{Cov}\,\left(\frac{\tilde{p}_{il}}{p_{il}}, \frac{\tilde{p}_{kl}}{p_{kl}}\right) = (C_1+C_2)\,\lambda_{ik}$

$\{\lambda_{ik}\}$

	1 "Scotia"	2 "Anton Dohrn"	3 "Willem Beukelsz"	4 "Dana"
1. "Scotia"	0.0620	-0.0190	-0.0418	-0.0012
2. "Anton Dohrn"	-0.0190	0.0263	-0.0023	-0.0050
3. "Willem Beukelsz"	-0.0418	-0.0023	0.0755	-0.0314
4. "Dana"	-0.0012	-0.0050	-0.0314	0.0376

(2) Density; $\mathrm{Var}\,\frac{D_{jl}}{D_{jl}} = (C_1+C_2)\,\mu_{jj}$; $\mathrm{Cov}\,\left(\frac{\tilde{D}_{jl}}{D_{jl}}, \frac{\tilde{D}_{kl}}{D_{kl}}\right) = (C_1+C_2)\,\mu_{jk}$ $(j \neq k)$

$\{\mu_{jk}\}$

Area	Area 1	Area 2	Area 3	Area 4	Area 5
1	0.0657	-0.0037	-0.0130	-0.0078	-0.0412
2	-0.0037	0.0966	-0.0256	-0.0136	-0.0538
3	-0.0130	-0.0256	0.0280	0.0109	-0.0002
4	-0.0078	-0.0136	0.0109	0.0277	-0.0173
5	-0.0412	-0.0538	-0.0002	-0.0173	0.1125

The χ^2 values in App. 3, Table 2 are approximate tests for rejecting the model. It is seen that the value is rather high for spring 1960. If we combine all χ^2 values we get the following pooled $(C_1 + C_2)$, estimated from the means:

$$\frac{\Sigma\chi^2 . \text{d. f}}{\Sigma \text{d.f}} = \frac{44.66 + 25.68 + 14.52 + 5.44}{2 + 6 + 4 + 2} = \frac{90.30}{14} = 6.45$$

and

$$\frac{\chi^2}{14} = \frac{6.45}{5.371} = 1.20 \qquad (\text{df} = 14) .$$

The test indicates fairly good agreement between data and model. In other words, the fishing power differences between ships are constant from area to area.

One thing has to be kept in mind. If the matrix (p, D) is a set of fishing powers and densities then $(kp, \frac{D}{k})$ is a set that will do just as well. We are not able to determine absolute values, only relative ones. But this means that we are unable to compare p's and D's between seasons without further information than just catch per effort. In App. 3, Table 3 are given the abundances in area 4 as found by Popp Madsen from catch and effort statistics for the Danish industrial fishery. If these abundance estimates are taken as comparable estimates we can find four k's ($k_1 = 1$, k_2, k_3, and k_4) thus:

$$1 . D_4^{S60} : k_2 . D_4^{A60} : k_3 . D_4^{S61} : k_4 . D_4^{A61}$$

$$= 3.67 : 3.14 : 1.31 : 6.36 ,$$

$$\text{or} \quad k_2 = 0.2058, \quad k_3 = 0.2936, \quad k_4 = 0.6019 ,$$

and by calculating $\frac{p}{k}$, k D comparable fishing powers and density estimates are obtained. The results are shown in App. 3, Table 4.

In App. 3, Table 5 are shown the age compositions as found from the cruises.

From App. 3, Tables 4 and 5 the age distributions have been determined for each area and cruise, and these are shown in App. 3, Table 6.

App. 3, Table 2 Values of χ^2 tests for comparison within ships and between ships and areas

Survey	d.f.	
Spring 1960	2	$\frac{\chi^2}{2} = \frac{(C_1 + C_2) \text{ estimated from means}}{(2.32)^2} = \frac{22.33}{5.371} = 4.16$
Autumn 1960	6	$\frac{\chi^2}{6} = \frac{(C_1 + C_2) \text{ estimated from means}}{(2.32)^2} = \frac{4.28}{5.371} = 0.80$
Spring 1961	4	$\frac{\chi^2}{4} = \frac{(C_1 + C_2) \text{ estimated from means}}{(2.32)^2} = \frac{3.63}{5.371} = 0.68$
Autumn 1961	2	$\frac{\chi^2}{2} = \frac{(C_1 + C_2) \text{ estimated from means}}{(2.32)^2} = \frac{2.72}{5.371} = 0.51$

App. 3, Table 3 Abundance estimates for area 4 from the "Bløden" fishery in number/100 hours

Survey	Abundance estimate (number/100 hours x 10^6)
Spring 1960 (March and April)	3.67
Autumn 1960 (September)	3.14
Spring 1961 (March and April)	1.31
Autumn 1961 (September)	6.36

App. 3, Table 4 Comparable fishing powers, densities and abundances

	Spring 1960	Autumn 1960	Spring 1961	Autumn 1961
Fishing power				
"Ernest Holt"	0.1396			
"Sir Lancelot"		2.341		
"Scotia"	2.251	0.6694	3.916	5.077
"Anton Dohrn"	15.38	3.902	3.272	3.593
"Willem Beukelsz"	0.5200	3.610	2.878	1.295
"Dana"		3.318	3.570	0.5121
Density				
Area 1	44.30	20.96	23.23	14.61
2	34.59	0.9471	122.3	6 871
3	2 262	153.3	123.2	418.5
4	412.6	353.0	147.3	715.1
5	870.7	0.3833	157.7	34.10
Abundance				
Area 1	2 702	1 279	1 417	891
2	346	9	1 223	68 710
3	61 074	4 139	3 326	11 300
4	13 616	11 649	4 861	23 598
5	18 285	8	3 312	716
Total	96 023	17 084	14 139	105 215
Standard deviation	47.6%	11.6%	29.9%	59.7%

App. 3, Table 5 Percentage age distributions by ships and areas, as caught

Survey	Area	Year-class 1960	1959	1958	1957	Others	Total number
S 60	1			23.03	23.65	53.32	28 036
	2			85.70	9.79	4.51	6 653
	3			22.55	55.99	21.46	21 855
	4			94.66	5.03	0.31	64 716
	5		0.05	93.44	4.43	2.08	6 544
A 60	1		0.13	14.12	31.58	54.17	1 558
	2			3.03	24.24	72.73	33
	3		4.12	7.86	36.78	51.24	34 271
	4		5.42	86.66	4.91	3.01	70 782
	5		32.00		4.00	64.00	75
S 61	1		40.16	7.85	9.60	42.39	9 949
	2		45.53	39.52	2.37	12.58	8 428
	3		33.74	40.84	15.71	9.71	12 811
	4		74.72	23.31	1.36	0.61	16 262
	5		31.34	56.43	5.38	6.85	11 610
A 61	1	27.95	26.70	3.01	9.02	33.24	1 363
	2	97.96	1.76	0.28			19 355
	3	0.45	3.26	43.16	19.25	33.88	55 970
	4	94.50	1.86	3.24	0.11	0.29	63 121
	5	97.96	1.13	0.68		0.23	884

App. 3, Table 6 Age distributions as abundances (fishing powers, densities and abundances, from App. 3, Tables 4 and 5)

Survey	Area	Year-class 1960	1959	1958	1957	Others	Total
S 60	1			622	639	1 441	2 702
	2			296	34	16	346
	3			13 772	34 195	13 106	61 073
	4			12 889	685	42	13 616
	5		9	17 085	810	380	18 284
			9	44 664	36 363	14 985	96 021
A 60	1		2	180	404	692	1 278
	2				2	7	9
	3		171	325	1 523	2 121	4 140
	4		631	10 095	572	351	11 649
	5		3			5	8
			807	10 600	2 501	3 176	17 084
S 61	1		569	111	136	601	1 417
	2		557	483	29	154	1 223
	3		1 122	1 358	522	323	3 325
	4		3 632	1 133	66	30	4 861
	5		1 038	1 868	178	227	3 311
			6 918	4 953	931	1 335	14 137
A 61	1	249	239	27	80	296	891
	2	67 308	1 209	192			68 709
	3	51	368	4 877	2 175	3 828	11 299
	4	22 300	439	765	26	68	23 598
	5	702	8	5		2	717
		90 610	2 263	5 866	2 281	4 194	105 214

The estimates found depend very much on the assumptions. To illustrate this, estimates have been determined based on another set of assumptions. As the basic assumption, we suppose that all ships have the same fishing power within the cruises. We shall still take the fishing to be random within the five areas. The model is now:

$$(c/f)_{ijl} = p_i \, D_{ij} + \Delta_{ijl} \tag{5}$$

where p is the fishing power, D the density and Δ a random component. i refers to the cruise, j to the area and l to the haul number. In App. 3, Table 7 are shown estimates of p.D from equation (5), using the catches per effort. In App. 3, Table 8 are shown the fishing powers and densities, taking area 4 densities identical with the values in App. 3, Table 4. Finally we have found the age distributions based on App. 3, Tables 5 and 8, and have given these figures in App. 3, Table 9.

App. 3, Tables 4 and 6 can be compared with App. 3, Tables 8 and 9 respectively. The great difference between the two models lies in the estimates of a_3 and a_5 in spring 1960 and of a_2 in autumn 1961.

The variances of the total abundances in App. 3, Tables 4 and 8 are estimated from the formulae:

$$\text{Total abundance} = T_j = \left[\alpha_i^5 \cdot \frac{\tilde{D}_{ij}}{D_{4j}} \right]_{i=1} G_j$$

where α_i is the number of squares per area i; then

$$T_j = \left(61 \cdot \frac{\tilde{D}_{1j}}{\tilde{D}_{4j}} + 10 \cdot \frac{\tilde{D}_{2j}}{\tilde{D}_{4j}} + 27 \cdot \frac{\tilde{D}_{3j}}{\tilde{D}_{4j}} + 33 + 21 \cdot \frac{\tilde{D}_{5j}}{\tilde{D}_{4j}}\right) G_j$$

$$\operatorname{Var}\left(\frac{\tilde{D}_{ij}}{\tilde{D}_{4j}}\right) \approx \left(\frac{\tilde{D}_{ij}}{\tilde{D}_{4j}}\right)^2 \cdot \left[\operatorname{Var}\left(\frac{\tilde{D}_{ij}}{D_{ij}}\right) + \operatorname{Var}\left(\frac{\tilde{D}_{4j}}{D_{4j}}\right) - 2\operatorname{cov}\left(\frac{\tilde{D}_{ij}}{D_{ij}}, \frac{\tilde{D}_{4j}}{D_{4j}}\right)\right]$$

$$\operatorname{cov}\left(\frac{\tilde{D}_{ij}}{\tilde{D}_{4j}}, \frac{\tilde{D}_{kj}}{\tilde{D}_{4j}}\right) \approx \frac{\tilde{D}_{ij}}{\tilde{D}_{4j}} \cdot \frac{\tilde{D}_{kj}}{\tilde{D}_{4j}} \cdot \left[\operatorname{cov}\left(\frac{\tilde{D}_{ij}}{D_{ij}}, \frac{\tilde{D}_{kj}}{D_{kj}}\right) - \operatorname{cov}\left(\frac{\tilde{D}_{ij}}{D_{ij}}, \frac{\tilde{D}_{4j}}{D_{4j}}\right) - \operatorname{cov}\left(\frac{\tilde{D}_{kj}}{D_{kj}}, \frac{\tilde{D}_{4j}}{D_{4j}}\right) + \operatorname{Var}\left(\frac{\tilde{D}_{4j}}{D_{4j}}\right)\right];$$

$$\operatorname{Var} G_j = 0$$

where i and k ($\neq 4$) refer to the area and j to the cruise; G is the relevant "Bløden" density. The variances are thus based on the assumption that the "Bløden" ratios are exact.

App. 3, Table 7 Density estimates under the assumption of no difference in fishing power between ships

Survey	Area	p.D	Var	sd	sd (%)
S 60	1	463.3	6 220	78.87	17
	2	532.2	52 112	228.3	43
	3	594.5	31 266	176.8	30
	4	2 660	1 556 686	1 248	47
	5	390.4	29 003	170.3	44
A 60	1	26.42	220.0	14.83	56
	2	3.143	3.23	1.797	57
	3	562.8	20 014	141.5	25
	4	1 276	224 397	473.7	37
	5	1.370	0.3852	0.6206	45
S 61	1	82.18	452.6	21.27	26
	2	411.1	34 816	186.6	45
	3	364.4	15 032	122.6	34
	4	508.2	140 744	375.1	74
	5	461.5	19 525	139.7	30
A 61	1	60.32	393.3	19.83	33
	2	3 519	9 150 803	3 025	86
	3	1 370	172 396	415.2	30
	4	2 004	660 895	813.0	41
	5	44.20	936.9	30.61	69

App. 3, Table 8 Comparable fishing powers, densities and abundances

	Spring 1960	Autumn 1960	Spring 1961	Autumn 1961
Fishing powers (all ships)	6.447	3.615	3.450	2.802
Density				
Area 1	71.86	7.309	23.82	21.52
2	82.55	0.8695	119.2	1 256
3	92.21	155.7	105.6	488.9
4	412.6	353.0	147.3	715.1
5	60.56	0.3790	133.8	15.77
Abundance				
Area 1	4 383	446	1 453	1 313
2	826	9	1 192	12 560
3	2 490	4 204	2 851	13 200
4	13 616	11 649	4 861	23 598
5	1 272	8	2 810	332
Total	22 587	16 316	13 167	51 003
Standard deviation	19.4%	12.5%	47.8%	31.4%

App. 3, Table 9 Age distributions as abundances (identical fishing powers)

Area	Year-class					Total
	1960	1959	1958	1957	Others	
1			1 009	1 037	2 337	4 383
2			708	81	37	826
3			561	1 394	534	2 489
4			12 889	685	42	13 616
5		1	1 189	56	26	1 272
		1	16 356	3 253	2 976	22 586
1		1	63	141	242	447
2				2	7	9
3		173	330	1 546	2 154	4 203
4		631	10 095	572	351	11 649
5			3		5	8
		805	10 491	2 261	2 759	16 316
1		584	114	139	616	1 453
2		543	471	28	150	1 192
3		962	1 164	448	277	2 851
4		3 632	1 133	66	30	4 861
5		881	1 586	151	192	2 810
		6 602	4 468	832	1 265	13 167
1	367	352	40	118	436	1 313
2	12 304	221	35			12 560
3	59	430	5 697	2 541	4 472	13 199
4	22 300	439	765	26	68	23 598
5	325	4	2		1	332
	35 355	1 446	6 539	2 685	4 977	51 002

Estimates of variances and covariances for the age compositions in App. 3, Tables 6 and 9 can be obtained. Here the following formulae are used:

$$\text{Total number of n-year-old fish} = T_j^n = \left[\sum_{i=1}^{5} \alpha_i \; p_{ij}^n \frac{\tilde{D}_{ij}}{\tilde{D}_{4j}} \right] G_j$$

$$T_j^n = \left[61 \cdot p_{1j}^n \cdot \frac{\tilde{D}_{1j}}{\tilde{D}_{4j}} + 10 \cdot p_{2j}^n \cdot \frac{\tilde{D}_{2j}}{\tilde{D}_{4j}} + 27 \cdot p_{3j}^n \cdot \frac{\tilde{D}_{3j}}{\tilde{D}_{4j}} + 33 \cdot p_{4j}^n \right.$$

$$\left. + 21 \cdot p_{5j}^n \cdot \frac{\tilde{D}_{5j}}{\tilde{D}_{4j}} \right] G_j$$

$$\mathrm{Var}\left(p_{ij}^n \cdot \frac{D_{ij}}{D_{4j}}\right) \approx (p_{ij}^n)^2 \cdot \mathrm{Var}\left(\frac{D_{ij}}{D_{4j}}\right) + \left(\frac{D_{ij}}{D_{4j}}\right)^2 \cdot \mathrm{Var}\,(p_{ij}^n)$$

$$\mathrm{Cov}\left(p_{ij}^n \frac{D_{ij}}{D_{4j}}, p_{ij}^m \frac{D_{ij}}{D_{4j}}\right) = p_{ij}^n \cdot p_{ij}^m \;\mathrm{Var}\left(\frac{D_{ij}}{D_{4j}}\right) + \left(\frac{D_{ij}}{D_{4j}}\right)^2 \cdot \mathrm{cov}\,(p_{ij}^n, p_{ij}^m)$$

$$\mathrm{Cov}\left(p_{ij}^n \frac{\tilde{D}_{ij}}{\tilde{D}_{4j}}, p_{kj}^m \frac{\tilde{D}_{kj}}{\tilde{D}_{4j}}\right) = p_{ij}^n \cdot p_{kj}^m \cdot \mathrm{cov}\left(\frac{\tilde{D}_{ij}}{\tilde{D}_{4j}}, \frac{\tilde{D}_{kj}}{D_{4j}}\right)$$

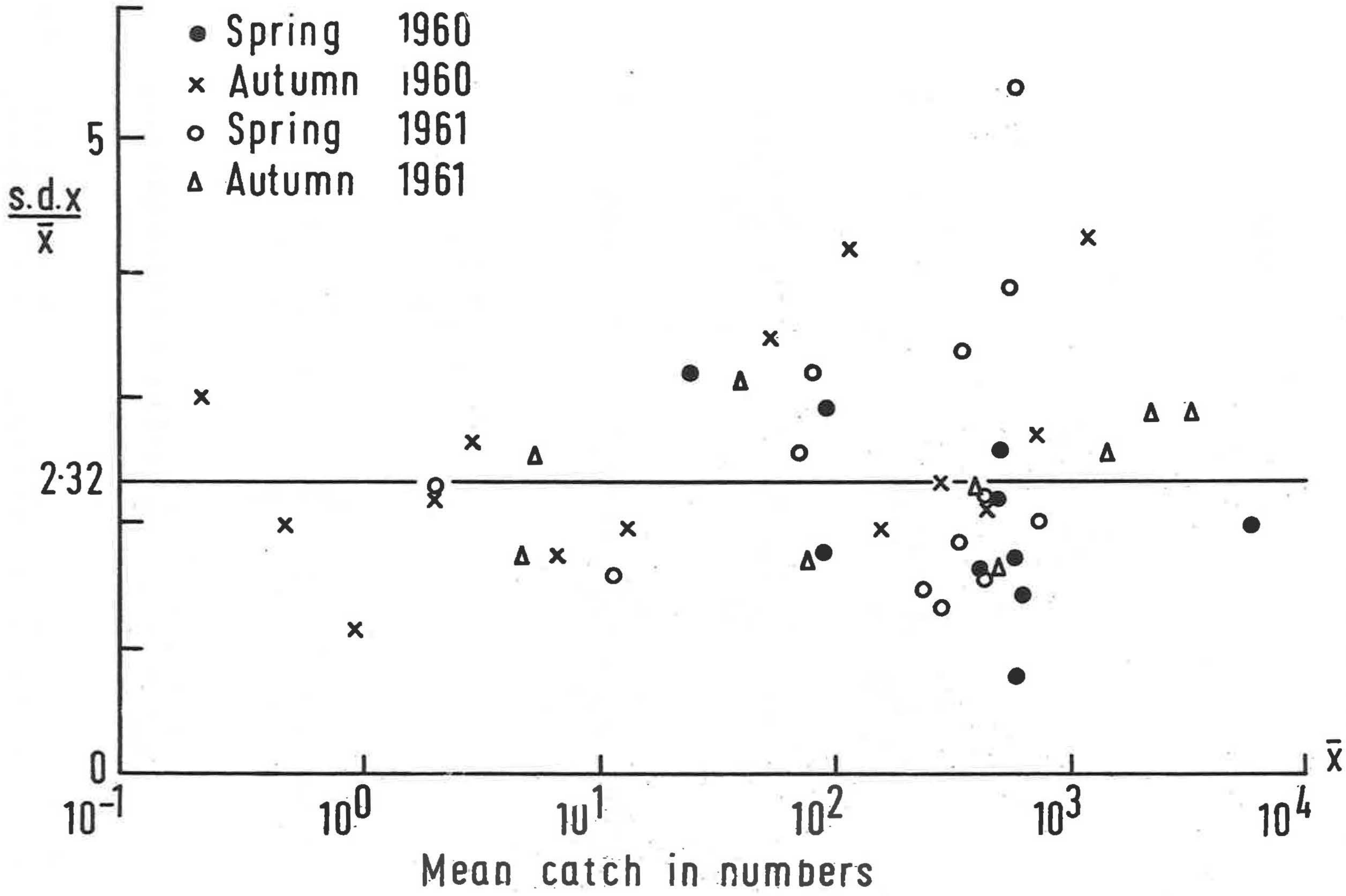
(i and k $\neq$ 4)

$$\mathrm{Var}\,(p_{ij}^n) = \frac{p_{ij}^n\,(1 - p_{ij}^n)}{\gamma_{ij}}$$

$$\mathrm{cov}\,(p_{ij}^n, p_{ij}^m) = -\frac{p_{ij}^n \; p_{ij}^m}{\gamma_{ij}}$$

where γ_{ij} = number of fish sampled for age and p_{ij} is the proportion of n-year-old fish in area i during j'th season, assuming the sampling for age to have been random.

The variances and covariances for the age distributions have not been calculated owing to lack of adequate help and calculating facilities.



App. 3, Figure 1 Relation between ratio of standard deviation of catch to mean catch on mean catch in numbers

Appendix 4 DISCRIMINANT FUNCTIONS

by K. P. Andersen, Danmarks Fiskeri- og Havundersøgelser, Charlottenlund

One of the important problems in North Sea herring research is the location of the different spawning stocks as young fish. Meristic characters have been used to distinguish between different stocks, but in a rough and rather subjective way. The trouble is that the available characters have distributions that overlap, and this makes it impossible to sort the fish directly. The problem is one of classification and it seems reasonable to use discriminant functions, a statistical tool developed by R. A. Fisher and precisely designed for handling classification problems.

The procedure will be illustrated by an artificial herring population: we suppose that we have a herring population consisting of three subpopulations and that the subpopulations differ by the distributions of l_1, K_2 and VS. We suppose further that the distributions in question are normal and that the three characters are stochastically independent within the three stocks. Finally we suppose that all differences between the subpopulations come from the means of the distributions, whereas the three variances for each character are identical.

In App. 4, Table 1 are given mean and variances for an artificial herring population consisting of the three subpopulations D, B and Bu.

App. 4, Table 1 Artificial Pure Stock characters

Subpopulation	Mean		
	l_1	K_2	VS
D	12.38	15.04	56.61
B	13.52	14.79	56.47
Bu	14.93	14.36	56.64
Variance	1.3400	0.7500	0.4200
Standard deviation	1.1576	0.8660	0.6481

We shall first see how to allocate a single fish to one of the three stocks, and shall do it stepwise.

1. Two stocks and one character

If a fish is known to belong either to D or B, and with l_1 as the only known character, we can choose a limit l_1^* and allocate according to the following rule:

$$\left.\begin{array}{l}\text{If the } l_1 \text{ of the fish} \geq l_1^* \text{ the fish is called B} \\ \text{If the } l_1 \text{ of the fish} \leq l_1^* \text{ the fish is called D}\end{array}\right\} \qquad R_1$$

The error of classifying a D as a B is

$$P_{D,B} = 1 - \phi\left(\frac{l_1^* - 12.38}{1.1576}\right)$$

(ϕ (a) is the probability that a standardized normal variate is $\leq$ a) and the error of classifying a B as a D is

$$P_{B,D} = \phi\left(\frac{l_1^* - 13.52}{1.1576}\right) \qquad \text{(see App. 4, Figure 1)}$$

If it is known that the fish is drawn at random from a population where the ratios of D and B are r_D and r_B ($r_D + r_B = 1$) then

$$\pi = P_{D,B} \cdot r_D + P_{B,D} \cdot r_B$$

is the total probability for misclassifying the fish, and a useful l_1^* can be found by minimizing π. In this way we get the best l_1^* in the sense that the total probability for misclassification is smallest. For $r_D = r_B = 0.5$ the best l_1^* is

$$l_1^* = \frac{\bar{l}_{1D} + \bar{l}_{1B}}{2} = 12.95 \quad \text{(bars indicate means)}$$

and as $\phi\left(\frac{12.95 - 12.38}{1.1576}\right) = 0.6886$ the total error of misclassification by using R_1 with this l_1^* is $= 0.3114$.

2. Two stocks and three characters

We now assume that all three characters are known and we will try to combine the three characters to a single character. We shall use a linear combination $X = a\, l_1 + b\, K_2 + c\, VS$. Any arbitrary set of a, b and c can be used, but some discriminate better than others. To find the X that discriminates best, we determine the a, b and c that gives the greatest relative difference in X between D and B, or that makes

$$\text{Discriminant} = D = \frac{\left[a(\bar{l}_{1B} - \bar{l}_{1D}) + b(\bar{K}_{2B} - \bar{K}_{2D}) + c(\overline{VS}_B - \overline{VS}_D)\right]^2}{\text{Var } X}$$

greatest possible (if they exist).

It can be shown that

$$a = k\frac{\bar{l}_{1B} - \bar{l}_{1D}}{\text{Var } l_1}, \quad b = k\frac{\bar{K}_{2B} - \bar{K}_{2D}}{\text{Var } K_2} \quad \text{and } c = k\frac{\overline{VS}_B - \overline{VS}_D}{\text{Var VS}}$$

maximizes D (k is an arbitrary constant $>$ 0 and in what follows we take k = 1). The combined character or discriminant function

$$X = \frac{l_{1B} - l_{1D}}{\text{Var } l_1} l_1 + \frac{K_{2B} - K_{2D}}{\text{Var } K_2} K_2 + \frac{VS_B - VS_D}{\text{Var VS}} VS$$

can now be used in the same way as we used l_1 under 1. In the example we find

$$X = 0.8507\, l_1 - 0.3333\, K_2 - 0.3333\, VS .$$

For means and variances we find:

$$\bar{X}_B = -12.2495 \qquad \bar{X}_D = -13.3493$$

$$(\text{Var } X)_B = (\text{Var } X)_D = \frac{(\bar{l}_{1B} - \bar{l}_{1D})^2}{(\text{Var } l_1)^2} \text{Var } l_1 +$$

$$\frac{(\bar{K}_{2B} - \bar{K}_{2D})^2}{(\text{Var } K_2)^2} \text{Var } K_2 + \frac{(\overline{VS}_B - \overline{VS}_D)^2}{(\text{Var VS})^2} \text{Var VS}$$

$$= \frac{(\bar{l}_{1B} - \bar{l}_{1D})}{\text{Var } l_1} (\bar{l}_{1B} - \bar{l}_{1D}) + \frac{(\bar{K}_{2B} - \bar{K}_{2D})}{\text{Var } K_2} (\bar{K}_{2B} - \bar{K}_{2D}) +$$

$$\frac{(\overline{VS}_B - \overline{VS}_D)}{\text{Var VS}} (\overline{VS}_B - \overline{VS}_D)$$

$$= \bar{X}_B - \bar{X}_D = 1.0998$$

If B and D have the same relative strength

$$X^* = \frac{\bar{X}_B + \bar{X}_D}{2} = -12.7994$$

is the best splitting point and we classify by using the rule:

$$\left.\begin{array}{l} D \text{ if } X \leq -12.7994 \\ B \text{ if } X > \ 12.7994 \end{array}\right\} \qquad R_2$$

As

$$\phi\left(\frac{X^* - \bar{X}_D}{\sqrt{Var X}}\right) = \phi\left(\frac{\frac{\bar{X}_B - \bar{X}_D}{2}}{\sqrt{\bar{X}_B - \bar{X}_D}}\right)$$

$$= \phi \frac{\sqrt{\bar{X}_B - \bar{X}_D}}{2} = \phi\,(0.5244) = 0.7000$$

the total error of misclassifying is

$$0.5 \times 0.3000 + 0.5 \times 3000 = 0.3000 \text{ or 30 per cent by using } R_2.$$

3. Three stocks and three characters

If our fish have all three classification possibilities, D, B and Bu, we proceed in this way.

We can calculate three discriminant functions:

$$(B/D) \quad X = \frac{\bar{l}_{1B} - \bar{l}_{1D}}{Var\, l_1} l_1 + \frac{\bar{K}_{2B} - \bar{K}_{2D}}{Var\, K_2} K_2 + \frac{\overline{VS}_B - \overline{VS}_D}{Var\, VS} VS \qquad (1)$$

$$(Bu/B) \quad Y = \frac{\bar{l}_{1Bu} - \bar{l}_{1B}}{Var\, l_1} l_1 + \frac{\bar{K}_{2Bu} - \bar{K}_{2B}}{Var\, K_2} K_2 + \frac{\overline{VS}_{Bu} - \overline{VS}_B}{Var\, VS} VS \qquad (2)$$

$$(Bu/D) \quad Z = \frac{\bar{l}_{1Bu} - \bar{l}_{1D}}{Var\, l_1} l_1 + \frac{\bar{K}_{2Bu} - \bar{K}_{2D}}{Var\, K_2} K_2 + \frac{\overline{VS}_{Bu} - \overline{VS}_D}{Var\, VS} VS \qquad (3)$$

Equations (1), (2) and (3) give:

$$Z = X + Y \qquad (4)$$

$$(Var\, X)_D = (Var\, X)_B = (Var\, X)_{Bu} = \bar{X}_B - \bar{X}_D$$

$$(Var\, Y)_D = (Var\, Y)_B = (Var\, Y)_{Bu} = \bar{Y}_{Bu} - \bar{Y}_B$$

$$(Var\, Z)_D = (Var\, Z)_B = (Var\, Z)_{Bu} = \bar{Z}_{Bu} - \bar{Z}_D$$

$$(Cov\,(X,Y))_D = (Cov\,(X,Y))_B = (Cov\, X,Y))_{Bu} = \bar{X}_{Bu} - \bar{X}_B = \bar{Y}_B - \bar{Y}_D$$

$$(\text{Cov}(X,Z))_D = (\text{Cov}(X,Z))_B = (\text{Cov}(X,Z))_{Bu} = \bar{X}_{Bu} - \bar{X}_D = \bar{Z}_B - \bar{Z}_D$$

$$(\text{Cov}(Y,Z))_D = (\text{Cov}(Y,Z))_B = (\text{Cov}(Y,Z))_{Bu} = \bar{Y}_{Bu} - \bar{Y}_D = \bar{Z}_{Bu} - \bar{Z}_B$$

Equation (4) supplies us with further formulae:

$$\text{Var } Z = \text{Var } X + \text{Var } Y + 2 \text{ Cov}(X,Y)$$

$$\text{Cov}(X,Z) = \text{Var } X + \text{Cov}(X,Y)$$

$$\text{Cov}(Y,Z) = \text{Cov}(X,Y) + \text{Var } Y.$$

For the hypothetical population this gives:

$$X = 0.8507\, l_1 - 0.3333\, K_2 - 0.3333\, VS$$

$$Y = 1.0522\, l_1 - 0.5733\, K_2 + 0.4048\, VS$$

$$Z = 1.9029\, l_1 - 0.9066\, K_2 + 0.715\, VS$$

with the means:

	$\bar{X}$	$\bar{Y}$	$\bar{Z}$
D	-13.3493	27.3195	13.9703
B	-12.2495	28.6057	16.3562
Bu	-10.9633	30.4046	19.4413

variances and covariances:

$$\text{Var } X = \bar{X}_B - \bar{X}_D = 1.0998$$

$$\text{Var } Y = \bar{Y}_{Bu} - \bar{Y}_B = 1.7989$$

$$\text{Cov}(X,Y) = \bar{X}_{Bu} - \bar{X}_B = 1.2862 = \bar{Y}_B - \bar{Y}_D = 1.2862$$

$$\text{Var } Z = \bar{Z}_{Bu} - \bar{Z}_D = 5.4710 = \text{Var } X + \text{Var } Y + 2 \text{ Cov}(X,Y) = 5.4711$$

$$\text{Cov}(X,Z) = \bar{X}_{Bu} - \bar{X}_D = 2.3860 = \bar{Z}_{Bu} - \bar{Z}_D = 2.3859$$

$$= \text{Var } X + \text{Cov}(X,Y) = 2.3860$$

$$\text{Cov}(Y,Z) = \bar{Y}_{Bu} - \bar{Y}_D = 3.0851 = \bar{Z}_{Bu} - \bar{Z}_B = 3.0851$$

$$= \text{Cov}(X,Y) + \text{Var } Y = 3.0851\,.$$

Three stocks supply three discriminant functions, but they are not independent. In fact two of them hold all the information, since any of them is a linear function of the two others ($Z = X + Y$, $X = Z - Y$, $Y = Z - X$). For each subpopulation (X, Y) is distributed as a two-dimensional normal distribution, and all differences between the distributions lie in the means.

When trying to classify a fish known to belong to one of two subpopulations we choose a splitting value of the discriminant function X. The analogous thing to do in the case of three subpopulations is to draw splitting curves (see (App. 4, Figure 2) and allocate the fish according to where its (X, Y) lies. The question of the existence of the best splitting curves is very complicated and will not be discussed here. The total probability of misclassification depends on the frequency of the three subpopulations and the splitting curves. In the case of equal frequencies the best splitting curves are given by

$$x = \frac{\bar{X}_D + \bar{X}_B}{2} = -12.7994$$

$$y = \frac{\bar{Y}_B + \bar{Y}_{Bu}}{2} = 29.5052$$

$$x + y = \frac{\bar{Z}_D + \bar{Z}_{Bu}}{2} = 16.7058$$

The splitting curves are straight lines and the rule for classification becomes:

$$\begin{array}{lll} \text{D}, & X \leq -12.7994 \text{ and } X + Y \leq 16.7058 & \\ \text{B}, & X > -12.7994 \text{ and } Y \leq 29.5052 & \text{(see App. 4, Figure 2)} \qquad R_3 \\ \text{Bu}, & X + Y > 16.7058 \text{ and } Y > 29.5052 & \end{array}$$

The correlation coefficients for X, Y and Z are:

$$\rho_{xy} = 0.9145$$

$$\rho_{xz} = 0.9727$$

$$\rho_{yz} = 0.9834$$

The misclassification probabilities can now be calculated. The probability for classifying a D as a B is the probability that $X \leq -12.7994$ and $Y > 29.5052$, when X and Y are normally distributed random variables with means -13.3493 and -12.2495, variances 1.0998 and 1.7989 and correlation coefficient 0.9145.

From the figures in App. 4, Table 1 and Tables VIII and IX in Pearson (1931) the following probabilities are found:

$P(D \text{ as } D) = P_{D,D} = 0.700$

$P(D \text{ as } B) = P_{D,B} = 0.249$

$P(D \text{ as } Bu) = P_{D,Bu} = 0.051$

$P(B \text{ as } D) = P_{B,D} = 0.300$

$P(B \text{ as } B) = P_{B,B} = 0.449$

$P(B \text{ as } Bu) = P_{B,Bu} = 0.251$

$P(Bu \text{ as } D) = P_{Bu,D} = 0.040$

$P(Bu \text{ as } B) = P_{Bu,B} = 0.211$

$P(Bu \text{ as } Bu) = P_{Bu,Bu} = 0.749$

When a fish is drawn at random from a population with the three subpopulations in the same proportions the probability of misclassification is:

$$\frac{1}{3} \cdot 0.249 + \frac{1}{3} \cdot 0.051 + \frac{1}{3} \cdot 0.300 + \frac{1}{3} \cdot 0.251 + \frac{1}{3} \cdot 0.040 + \frac{1}{3} \cdot 0.211 = 0.367.$$

Let us try to classify a fish with the characters:

$l_1 : 14.5, \quad K_2 : 14, \quad VS : 57.$

1. Let it be given that the fish belongs to D or B. Using l_1 alone and R_1 with $l_1^* = 12.95$ we classify the fish as a B.
   Using all three characters we find $X = -11.33$ and by R_2 we classify the fish as a B.

2. Let it be given that all three stocks are present. We calculate $Y = 30.30$ and $Z = 18.97$, and by R_3 we classify the fish as a Bu.

Estimates of the true proportions

In App. 4, Table 2 are given meristic characters for an artificial sample (constructed by using random numbers) of a mixture of D, B and Bu.[/] We wish to estimate the relative abundances n_D, n_B and n_{Bu} of the three stocks.

[/] The sample consists actually of 30 D (1-30), 20 B (31-50) and 50 Bu (51-100) and we shall take it as a random sample from a population with $n_D = 0.3$, $n_B = 0.2$ and $n_{Bu} = 0.5$. This is of course not the case, but it is all right for illustration.

App. 4, Table 2 Artificial mixture of three Pure Stocks

Fish no.	l_1	K_2	VS	Fish no.	l_1	K_2	VS	Fish no.	l_1	K_2	VS	Fish no.	l_1	K_2	VS
1	14.5	14	57	26	11.5	16	57	51	14.5	14	56	76	16.5	14	56
2	13.5	16	57	27	13.0	16	56	52	14.0	15	57	77	13.5	15	56
3	11.0	17	58	28	10.5	15	57	53	15.0	14	56	78	14.5	15	57
4	12.0	14	56	29	11.0	15	57	54	14.5	14	57	79	15.5	15	56
5	11.5	15	57	30	13.0	16	57	55	16.0	14	57	80	15.0	14	58
6	11.0	14	57	31	13.5	15	56	56	15.5	15	57	81	13.0	14	58
7	12.5	15	57	32	15.0	14	56	57	14.5	15	55	82	14.5	13	56
8	13.5	15	57	33	15.0	14	56	58	16.5	14	57	83	14.0	15	57
9	13.5	16	56	34	13.5	16	57	59	14.5	15	56	84	17.5	15	57
10	12.0	14	57	35	13.0	15	56	60	14.0	15	57	85	16.5	15	56
11	14.0	15	57	36	13.5	14	56	61	15.0	14	56	86	16.5	15	57
12	13.0	16	57	37	13.0	15	56	62	16.0	13	57	87	14.5	14	57
13	10.5	14	57	38	12.5	15	57	63	14.5	14	57	88	13.0	15	57
14	12.0	15	57	39	11.5	15	56	64	14.0	14	58	89	14.0	13	57
15	13.0	14	58	40	11.0	14	56	65	16.0	14	56	90	13.5	13	56
16	13.0	16	56	41	12.0	13	57	66	14.0	12	56	91	14.5	14	57
17	12.5	15	57	42	15.5	15	56	67	17.5	14	56	92	16.5	15	56
18	15.0	15	58	43	15.5	14	56	68	14.5	16	57	93	14.0	13	57
19	11.0	15	57	44	12.0	14	57	69	15.5	13	56	94	15.5	15	57
20	11.5	16	56	45	13.5	15	56	70	14.5	14	56	95	14.5	14	56
21	13.0	15	56	46	15.0	15	56	71	15.0	15	57	96	16.0	14	57
22	12.5	16	58	47	13.5	16	58	72	15.0	16	56	97	15.0	14	57
23	12.5	15	57	48	13.5	14	56	73	14.0	14	57	98	15.0	15	57
24	12.0	16	58	49	14.5	14	56	74	13.0	14	56	99	14.5	14	57
25	11.0	15	57	50	13.0	14	56	75	14.5	13	57	100	15.5	15	57

However, let us first try to estimate the relative abundances from a sample drawn from a population consisting of two stocks, say D and B. The sample can consist of the first 50 fish of App. 4, Table 2. We start by calculating X (shown in App. 4, Table 3) and classify accordingly to R_2 (shown in App. 4, Table 3 as C_1). In this way we get the following estimates of the two relative abundances n_D and n_B:

$$n^*_D = \frac{31}{50} = 0.62$$

and $$n^*_B = \frac{19}{50} = 0.38$$

As $$\bar{n}^*_D = n_D \; P_{D,D} + n_B \; P_{B,D} \qquad (5)$$

and $$\bar{n}^*_B = n_D \; P_{D,B} + n_B \; P_{B,B} \qquad (6)^{/}$$

it is seen that the mean of the estimates n^* depends on n_D, n_B and X^* and thus generally are biased. From (5) and $n_D + n_B = 1$ we get

$$n_D = \frac{\bar{n}^*_D - P_{B,D}}{P_{D,D} - P_{B,D}}$$

and $$n_B = \frac{P_{D,D} - \bar{n}^*_D}{P_{D,D} - P_{B,D}}$$

and this gives us the unbiased estimates:

$$\hat{n}_D = \frac{n^*_D - P_{B,D}}{P_{D,D} - P_{B,D}} \qquad (7)$$

and $$n_B = \frac{P_{D,D} - n^*_D}{P_{D,D} - P_{B,D}} = 1 - \hat{n}_D \qquad (8)$$

If N is the sample size and if $N_{B,D}$ represents the number of B fish classified by R_2 as D etc. we have:

/ The P's are the classification probabilities from page 4 , belonging to X alone, and not the values from page 7 , which belong to X and Y.

$$n^*_D = \frac{N_{D,D} + N_{B,D}}{N} \quad \text{and} \quad n^*_B = \frac{N_{D,B} + N_{B,B}}{N},$$

and n^*_D , n^*_B are bionomially distributed, with means (7) and (8) and variances and covariances:

$$\mathrm{Var}\,(n^*_D) = \frac{1}{N}\,\overline{n^*_D}\,(1 - \overline{n^*_D})$$

$$\mathrm{Var}\,(n^*_B) = \frac{1}{N}\,\overline{n^*_B}\,(1 - \overline{n^*_B})$$

$$\mathrm{Cov}\,(n^*_D, n^*_B) = -\frac{1}{N}\,\overline{n^*_D}\;\overline{n^*_B} \quad \text{etc.}$$

In our example we find:

$$P_{D,D} = 0.7000\,; \quad P_{B,D} = 0.3000$$

$$P_{D,D} - P_{B,D} = 0.4000$$

$$\hat{n}_D = \frac{0.6200 - 0.3000}{0.4000} = 0.80$$

$$\hat{n}_B = \frac{0.7000 - 0.6200}{0.4000} = 0.20$$

$$\overline{n^*_D} = 0.6 \times 0.7 + 0.4 \times 0.3 = 0.54$$

(see note on page 9)

We can examine the gain by using the information from K_2 and VS by finding the variances when l_1 is used alone.

In this case we have used R_1 with $l^*_1 = 12.95$

$$P_{D,D} = 0.6886 \qquad P_{B,D} = 0.3114$$

$$P_{D,D} - P_{B,D} = 0.3772$$ (for the P's, see page 2)

$$n^*_D = 0.54 \qquad n^*_B = 0.46$$

$$\hat{n}_D = \frac{0.5400 - 0.3114}{0.3772} = 0.6060$$

$$n_B = \frac{0.6886 - 0.5400}{0.3772} = 0.3940$$

$$\text{Mean} = E(n_D^*) = 0.53772$$

$$\text{Var}(n_D^*) = 0.004972$$

$$\text{Var}(n_D) = 0.03495$$

The gain is thus $\frac{0.03495 - 0.03105}{0.03495} \times 100 = 11.2$ per cent.

Let us then take all 100 fish of App. 4, Table 2 and work with three stocks. The thing to do now is to calculate Y and Z, and the results of the calculations are shown in App. 4, Table 3. We then apply R_3 and get the classifications shown as C_2 in App. 4, Table 3, and we get the following estimates of the three abundances:

$$n_D^* = 0.33$$

$$n_B^* = 0.22$$

$$n_{Bu}^* = 0.45$$

We now proceed by using the two-stock case as a model:

As
$$\overline{n_D^*} = n_D P_{D,D} + n_B P_{B,D} + n_{Bu} P_{Bu,D} \tag{9}$$

$$\overline{n_B^*} = n_D P_{D,B} + n_B P_{B,B} + n_{Bu} P_{Bu,B} \tag{10}$$

and
$$\overline{n_{Bu}^*} = n_D P_{D,Bu} + n_B P_{B,Bu} + n_{Bu} P_{Bu,Bu} \tag{11}$$

We see that the expectations depend on the n's and the classification limits, and the n*'s are thus biased in general. Equations (9) and (10) and $n_D + n_B + n_{Bu} = 1$ give us the following unbiased estimates of the true proportions:

$$\hat{n}_D = \frac{D_D}{D} ; \quad \hat{n}_B = \frac{D_B}{D} ; \quad \hat{n}_{Bu} = \frac{D_{Bu}}{D} ;$$

where

App. 4, Table 3 Values of X, Y and Z and stock assignment for the individuals of App. 4, Table 2

Fish no.	X	Y	Z	C_1	C_2
1	-11.33	30.30	18.97	B	Bu
2	-12.85	28.11	15.26	D	D
3	-15.64	25.31	9.67	D	D
4	-13.12	27.27	14.15	D	D
5	-14.22	26.57	12.35	D	D
6	-14.31	26.62	12.31	D	D
7	-13.36	27.62	14.26	B	D
8	-12.51	28.68	16.17	B	B
9	-12.51	27.70	15.19	B	B
10	-13.46	27.67	14.21	D	D
11	-12.09	29.20	17.11	B	B
12	-13.27	27.58	14.31	D	D
13	-14.73	26.10	11.37	D	D
14	-13.79	27.10	13.31	D	D
15	-12.94	29.13	16.19	D	D
16	-12.94	27.18	14.24	D	D
17	-13.36	27.63	14.27	D	D
18	-11.57	30.66	19.09	B	Bu
19	-14.64	26.05	11.41	D	D
20	-14.22	25.60	11.38	D	D
21	-12.61	27.75	15.14	B	B
22	-14.03	27.46	13.43	D	D
23	-13.36	27.63	14.27	D	D
24	-14.46	26.93	12.47	D	D
25	-14.64	26.05	11.41	D	D
26	-14.55	26.00	11.45	D	D
27	-12.94	27.18	14.24	D	D
28	-15.07	25.52	10.45	D	D
29	-14.64	26.05	11.41	D	D
30	-13.27	27.58	14.31	D	D
31	-12.18	28.27	16.09	B	B
32	-10.57	30.43	19.86	B	Bu
33	-10.57	30.43	19.86	B	Bu
34	-12.85	28.11	15.26	D	D
35	-12.61	27.75	15.14	B	B
36	-11.85	28.85	17.00	B	B
37	-12.61	27.75	15.14	B	B
38	-13.36	27.63	14.27	D	D
39	-13.88	26.17	12.29	D	D
40	-13.97	26.22	12.25	D	D
41	-13.12	28.25	15.13	D	D
42	-10.48	30.38	19.90	B	Bu
43	-10.14	30.95	20.81	B	Bu
44	-13.46	27.67	14.21	D	D
45	-12.18	28.27	16.09	B	B
46	-10.90	29.85	18.95	B	Bu
47	-13.18	28.51	15.33	D	D
48	-11.85	28.85	17.00	B	B
49	-11.00	29.90	18.90	B	Bu
50	-12.27	28.32	16.05	B	B

App. 4, Table 3 continued

Fish no.	X	Y	Z	C_2
51	-11.00	29.90	18.90	Bu
52	-12.09	29.20	17.11	B
53	-10.57	30.43	19.86	Bu
54	-11.33	30.30	18.97	Bu
55	-10.05	31.88	21.83	Bu
56	-10.81	30.78	19.97	Bu
57	-11.00	28.92	17.92	B
58	- 9.63	32.41	22.78	Bu
59	-11.33	29.33	18.00	B
60	-12.09	29.20	17.11	B
61	-10.57	30.43	19.86	Bu
62	- 9.72	32.46	22.74	Bu
63	-11.33	30.30	18.97	Bu
64	-12.09	30.18	18.09	Bu
65	- 9.72	31.48	21.76	Bu
66	-10.76	30.52	19.76	Bu
67	- 8.44	33.06	24.62	Bu
68	-12.00	29.16	17.16	B
69	- 9.81	31.52	21.71	Bu
70	-11.00	29.90	18.90	Bu
71	-11.24	30.26	19.02	Bu
72	-11.24	29.28	18.04	B
73	-11.75	29.78	18.03	Bu
74	-12.27	28.32	16.05	B
75	-11.00	30.88	19.88	Bu
76	- 9.29	32.00	22.71	Bu
77	-12.18	28.27	16.09	B
78	-11.66	29.73	18.07	Bu
79	-10.48	30.38	19.90	Bu
80	-11.24	31.24	20.00	Bu
81	-12.94	29.13	16.19	D
82	-10.66	30.47	19.81	Bu
83	-12.09	29.20	17.11	B
84	- 9.11	32.89	23.78	Bu
85	- 9.63	31.43	21.80	Bu
86	- 9.96	31.84	21.88	Bu
87	-11.33	30.30	18.97	Bu
88	-12.94	28.15	15.21	D
89	-11.42	30.35	18.93	Bu
90	-11.51	29.42	17.91	B
91	-11.33	30.30	18.97	Bu
92	- 9.63	31.43	21.80	Bu
93	-12.09	29.20	17.11	B
94	-10.81	30.78	19.97	Bu
95	-11.00	29.90	18.90	Bu
96	-10.05	31.88	21.83	Bu
97	-10.90	30.83	19.93	Bu
98	-11.24	30.26	19.02	Bu
99	-11.33	30.30	18.97	Bu
100	-10.81	30.78	19.97	Bu

$$\text{Determinant} = D = P_{D,D}\, P_{B,B} + P_{B,D}\, P_{Bu,B} + P_{D,B}\, P_{Bu,D}$$

$$- P_{B,B}\, P_{Bu,D} - P_{D,B}\, P_{B,D} - P_{D,D}\, P_{Bu,B}$$

$$D_D = n_D^*\, P_{B,B} + P_{B,D}\, P_{Bu,B} + n_B^*\, P_{Bu,D}$$

$$- P_{B,B}\, P_{Bu,D} - n_B^*\, P_{B,D} - n_D^*\, P_{Bu,B}$$

$$= a\, n_D^* + b\, n_B^* + c \qquad (12)$$

$$D_B = n_B^*\, P_{B,D} + n_D^*\, P_{Bu,B} + P_{D,B}\, P_{Bu,D}$$

$$- n_B^*\, P_{Bu,D} - n_D^*\, P_{D,B} - P_{D,D}\, P_{Bu,B}$$

$$= d\, n_D^* + e\, n_B^* + f \qquad (13)$$

$$D_{Bu} = P_{D,D}\, P_{B,B} + P_{B,D}\, n_B^* + P_{D,B}\, n_D^*$$

$$- P_{B,B}\, n_D^* - P_{D,B}\, P_{B,D} - P_{D,D}\, n_B^*$$

$$= g\, n_D^* + h\, n_B^* + i \qquad (14)$$

(n_D^*, n_B^*, n_{Bu}^*) is multinormally distributed, with means (9), (10) and (11) and variances and covariances

$$\mathrm{Var}\,(n_D^*) = \frac{1}{N}\, \overline{n_D^*}\, (1 - \overline{n_D^*})$$

$$\mathrm{Var}\,(n_B^*) = \frac{1}{N}\, \overline{n_B^*}\, (1 - \overline{n_B^*})$$

$$\mathrm{Var}\,(n_{Bu}^*) = \frac{1}{N}\, \overline{n_{Bu}^*}\, (1 - \overline{n_{Bu}^*})$$

$$\mathrm{Cov}\,(n_D^*,\, n_B^*) = -\frac{1}{N}\, \overline{n_D^*}\, \overline{n_B^*}$$

$$\mathrm{Cov}\,(n_D^*,\, n_{Bu}^*) = -\frac{1}{N}\, \overline{n_D^*}\, \overline{n_{Bu}^*}$$

$$\text{Cov}\,(n^*_B\,,\,n^*_{Bu}) = -\frac{1}{N}\,\overline{n^*_B}\,\overline{n^*_{Bu}}$$

The variances of the unbiased estimates ($\hat{n}_D$, $\hat{n}_B$, $\hat{n}_{Bu}$) of n_D, n_B and n_{Bu} are

$$\text{Var}\,(\hat{n}_D) = \frac{\text{Var}\,(D_D)}{D^2} \quad \text{etc.}$$

From (12), (13) and (14) we may write:

$$\text{Var}\,(n_D) = \frac{1}{D^2}\left[a^2\,\text{Var}\,(n^*_D) + b^2\,\text{Var}\,(n^*_B) - 2\,a\,b\,\text{Cov}\,(n^*_D\,,\,n^*_B)\right] \quad \text{etc.}$$

$$\text{Cov}\,(\hat{n}_D\,,\,\hat{n}_B) = \frac{1}{D^2}\left[(a\,d\,\text{Var}\,(n^*_D) + b\,e\,\text{Var}\,(n^*_B) + (a\,e + b\,d)\,\text{Cov}\,(n^*_D, n^*_B)\right]$$

etc.

For the constructed stock given in App. 4, Table 1 we find the following estimates of the true abundance:

$$\hat{n}_D = \frac{0.238\,n^*_D - 0.260\,n^*_B + 0.04534}{0.1472}\,,$$

$$\hat{n}_B = \frac{-0.038\,n^*_D + 0.660\,n^*_B - 0.13774}{0.1472}\,,$$

and $$\hat{n}_{Bu} = \frac{-0.200\,n^*_D - 0.400\,n^*_B + 0.23960}{0.1472}$$

(see page 7).

If $n_D = 0.3$, $n_B = 0.2$, and $n_{Bu} = 0.5$ we get the following means, variances and covariances:

$E\,(n^*_D) = 0.29$, $E\,(n^*_B) = 0.27$, $E\,(n^*_{Bu}) = 0.44$;

$\text{Var}\,(n^*_D) = 0.002059$, $\text{Var}\,(n^*_B) = 0.001971$, $\text{Var}\,(n^*_{Bu}) = 0.002464$.

$\text{Cov}\,(n^*_D, n^*_B) = -0.000783$, $\text{Cov}\,(n^*_D, n^*_{Bu}) = -0.001276$, $\text{Cov}\,(n^*_B, n^*_{Bu}) = -0.001188$.

$\mathrm{Var}(\hat{n}_D) = 0.01600$, $\mathrm{Var}(\hat{n}_B) = 0.04157$, $\mathrm{Var}(\hat{n}_{Bu}) = 0.01257$.

$\mathrm{Cov}(\hat{n}_D, \hat{n}_B) = -0.02250$, $\mathrm{Cov}(\hat{n}_D, \hat{n}_{Bu}) = 0.00650$, $\mathrm{Cov}(\hat{n}_B, \hat{n}_{Bu}) = -0.01907$

for a sample of 100 fish.

Taking App. 4, Table 2 as an example of such a sample we find:

$$\hat{n}_D = 0.4530 \qquad \text{standard deviation} = 0.1265$$

$$\hat{n}_B = -0.0345 \qquad \text{standard deviation} = 0.2039$$

$$\hat{n}_{Bu} = 0.5815 \qquad \text{standard deviation} = 0.1121$$

We can again find the gain by adding K_2 and VS. The adequate classification rule when using l_1 alone is:

$$\left.\begin{array}{ll} \text{D} & l_1 \leq \dfrac{12.38 + 13.52}{2} = 12.9500 \\[2ex] \text{B} & 12.9500 < l_1 \leq \dfrac{13.52 + 14.93}{2} = 14.2250 \\[2ex] \text{Bu} & 14.2250 < l_1 \end{array}\right\} \quad R_4$$

R_4 gives

$n^*_D = 0.23$, $n^*_B = 0.32$, $n^*_{Bu} = 0.45$.

$\hat{n}_D = 0.0929$, $\hat{n}_B = 0.4726$, $\hat{n}_{Bu} = 0.4345$.

$\mathrm{Var}(\hat{n}_D) = 0.02249$, $\mathrm{Var}(\hat{n}_B) = 0.06485$, $\mathrm{Var}(\hat{n}_{Bu}) = 0.01815$.

$\mathrm{Cov}(\hat{n}_D, \hat{n}_B) = -0.03460$, $\mathrm{Cov}(\hat{n}_D, \hat{n}_{Bu}) = 0.01210$, $\mathrm{Cov}(\hat{n}_B, \hat{n}_{Bu}) = -0.03025$.

The gain is

$$\text{for D} = 100 \times \frac{0.02249 - 0.01600}{0.02249} = 28.9 \text{ per cent}$$

$$\text{for B} = 100 \times \frac{0.06485 - 0.04157}{0.06485} = 35.9 \text{ per cent}$$

$$\text{for Bu} = 100 \times \frac{0.01815 - 0.01257}{0.01815} = 30.7 \text{ per cent}$$

We find for different sample sizes of mixed fish the following standard deviations:

Sample size	Standard deviation of	Using l_1	Using $l_1/K_2/VS$
100 fish	$\hat{n}_D$	0.1500	0.1265
	$\hat{n}_B$	0.2537	0.2039
	$\hat{n}_{Bu}$	0.1347	0.1121
1 000 fish	$\hat{n}_D$	0.0474	0.0400
	$\hat{n}_B$	0.0802	0.0645
	$\hat{n}_{Bu}$	0.0426	0.0355
5 000 fish	$\hat{n}_D$	0.0212	0.0179
	$\hat{n}_B$	0.0359	0.0288
	$\hat{n}_{Bu}$	0.0190	0.0159

To illustrate the sort of results that can be expected 50 random samples of 100 fish from a population consisting of 30 per cent D, 20 per cent B, and 50 per cent Bu have been constructed, and the result is shown in App. 4, Table 4, giving the preliminary classification. In App. 4, Table 5 are shown the unbiased estimates of the n's, together with the estimates based on 1 000 and 5 000 fish.

The Pure Stock l_1 bias and the iterative method

If the mean l_1 values are determined from older fish it can be expected that the means are biased. We shall suppose that:

$\bar{l}^*_{1\,D}$ = Mean l_1 for D as determined from older fish $= \bar{l}_{1\,D} + \Delta$

= true Mean l_1 for D plus an unknown constant

$$\bar{l}^*_{1\,B} = \bar{l}_{1\,B} + \Delta$$

$$\bar{l}^*_{1Bu} = \bar{l}_{1\,Bu} + \Delta$$

We can still determine the true discriminant function as

$$\bar{l}^*_{1\,B} - \bar{l}^*_{1\,D} = \bar{l}_{1\,B} - \bar{l}_{1\,D}$$

but we are not able to find the splitting curves used in R_3.

App. 4, Table 4 Preliminary classification, n*, of 50 random samples of known mixtures of three stocks.

N = 100

Sample no.	n^*_D	n^*_B	n^*_{Bu}	Sample no.	n^*_D	n^*_B	n^*_{Bu}
1	0.32	0.26	0.42	29	0.25	0.35	0.40
2	0.29	0.25	0.46	30	0.31	0.25	0.44
3	0.25	0.31	0.44	31	0.28	0.28	0.44
4	0.33	0.21	0.46	32	0.25	0.33	0.42
5	0.23	0.24	0.53	33	0.25	0.28	0.47
6	0.36	0.31	0.33	34	0.28	0.23	0.49
7	0.20	0.33	0.47	35	0.29	0.26	0.45
8	0.30	0.30	0.40	36	0.28	0.29	0.43
9	0.28	0.24	0.48	37	0.31	0.32	0.37
10	0.27	0.26	0.47	38	0.35	0.23	0.42
11	0.30	0.28	0.42	39	0.33	0.28	0.39
12	0.27	0.23	0.50	40	0.27	0.26	0.47
13	0.26	0.28	0.46	41	0.29	0.31	0.40
14	0.33	0.25	0.42	42	0.36	0.29	0.35
15	0.30	0.23	0.47	43	0.25	0.23	0.52
16	0.31	0.32	0.37	44	0.35	0.25	0.40
17	0.33	0.26	0.41	45	0.26	0.29	0.45
18	0.22	0.27	0.51	46	0.23	0.29	0.48
19	0.24	0.28	0.48	47	0.34	0.26	0.40
20	0.30	0.26	0.44	48	0.32	0.19	0.49
21	0.36	0.25	0.39	49	0.24	0.29	0.47
22	0.22	0.31	0.47	50	0.24	0.37	0.39
23	0.29	0.30	0.41				
24	0.31	0.26	0.43				
25	0.31	0.26	0.43	Mean	0.2850	0.2760	0.4390
26	0.21	0.34	0.45				
27	0.27	0.34	0.39				
28	0.26	0.24	0.50	Theoretical mean	0.2900	0.2700	0.4400

$\mathrm{Var}\,(n^*_D) = 0.002059$ Standard deviation = 0.0454

$\mathrm{Var}\,(n^*_B) = 0.001971$ Standard deviation = 0.0444

$\mathrm{Var}\,(n^*_{Bu}) = 0.002464$ Standard deviation = 0.0496

App. 4, Table 4 (continued)

N = 1000

Sample no.	n^*_D	n^*_B	n^*_{Bu}
1-10	0.283	0.271	0.446
11-20	0.286	0.266	0.448
21-30	0.279	0.290	0.431
31-40	0.289	0.276	0.435
41-50	0.288	0.277	0.435
Mean	0.2850	0.2760	0.4390
Theoretical mean	0.2900	0.2700	0.4400

$\mathrm{Var}(n^*_D) = 0.0002059$ Standard deviation = 0.0143

$\mathrm{Var}(n^*_B) = 0.0001971$ Standard deviation = 0.0140

$\mathrm{Var}(n^*_{Bu}) = 0.0002464$ Standard deviation = 0.0157

N = 5000

Sample no.	n^*_D	n^*_B	n^*_{Bu}
1-50	0.2850	0.2760	0.4390
Theoretical mean	0.2900	0.2700	0.4400

$\mathrm{Var}(n^*_D) = 0.00004118$ Standard deviation = 0.0064

$\mathrm{Var}(n^*_B) = 0.00003942$ Standard deviation = 0.0063

$\mathrm{Var}(n^*_{Bu}) = 0.00004928$ Standard deviation = 0.0070

App. 4, Table 5 Unbiased estimates, $\hat{n}$, of the 50 random samples of known mixtures of three stocks (see App. 4, Table 4)

Sample no.	$\hat{n}_D$	$\hat{n}_B$	$\hat{n}_{Bu}$	Sample no.	$\hat{n}_D$	$\hat{n}_B$	$\hat{n}_{Bu}$
1	0.3662	0.1474	0.4864	31	0.2662	0.2474	0.4864
2	0.3353	0.1103	0.5543	32	0.1293	0.4793	0.3913
3	0.1647	0.5897	0.4457	33	0.2177	0.2552	0.5272
4	0.4707	-0.0793	0.6087	34	0.3545	0.0232	0.6223
5	0.2560	0.0810	0.6630	35	0.3177	0.1552	0.5272
6	0.3425	0.3613	0.2962	36	0.2485	0.2923	0.4592
7	0.0485	0.4923	0.4592	37	0.2440	0.4190	0.3370
8	0.2632	0.3319	0.4049	38	0.4677	0.0052	0.5272
9	0.3368	0.0681	0.5951	39	0.3470	0.2345	0.4185
10	0.2853	0.1603	0.5543	40	0.2853	0.1603	0.5543
11	0.2985	0.2423	0.4592	41	0.2293	0.3793	0.3913
12	0.3383	0.0258	0.6359	42	0.3779	0.2716	0.3505
13	0.2338	0.2526	0.5136	43	0.3060	0.0310	0.6630
14	0.4000	0.1000	0.5000	44	0.4323	0.0948	0.4728
15	0.3868	0.0181	0.5951	45	0.2162	0.2974	0.4864
16	0.2440	0.4190	0.3370	46	0.1677	0.3052	0.5272
17	0.3823	0.1448	0.4728	47	0.3985	0.1423	0.4592
18	0.1868	0.2181	0.5951	48	0.4898	-0.1664	0.6766
19	0.2015	0.2577	0.5408	49	0.1838	0.3026	0.5136
20	0.3338	0.1526	0.5136	50	0.0425	0.6613	0.2962
21	0.4485	0.0923	0.4592				
22	0.1162	0.3974	0.4864				
23	0.2470	0.3345	0.4185	Mean	0.2813	0.2282	0.4905
24	0.3500	0.1500	0.5000				
25	0.3500	0.1500	0.5000				
26	0.0470	0.5345	0.4185				
27	0.1440	0.5190	0.3370				
28	0.3045	0.0732	0.6223	Theoretical mean	0.3000	0.2000	0.5000
29	0.0940	0.5690	0.3370				
30	0.3677	0.1052	0.5272				

Var $(\hat{n}_D)$ = 0.01600 Standard deviation = 0.1265

Var $(\hat{n}_B)$ = 0.04157 Standard deviation = 0.2039

Var $(\hat{n}_{Bu})$ = 0.01257 Standard deviation = 0.1121

App. 4, Table 5 (continued)

Sample no.	$\hat{n}_D$	$\hat{n}_B$	$\hat{n}_{Bu}$
1-10	0. 2869	0. 2063	0. 5068
11-20	0. 3006	0. 1831	0. 5163
21-30	0. 2469	0. 2925	0. 4606
31-40	0. 2878	0. 2272	0. 4851
41-50	0. 2844	0. 2319	0. 4837
Mean	0. 2813	0. 2282	0. 4905
Theoretical mean	0. 3000	0. 2000	0. 5000

Var ($\hat{n}_D$) = 0. 001600 Standard deviation = 0. 0400

Var ($\hat{n}_B$) = 0. 004157 Standard deviation = 0. 0645

Var ($\hat{n}_{Bu}$) = 0. 001257 Standard deviation = 0. 0355

Sample no.	$\hat{n}_D$	$\hat{n}_B$	$\hat{n}_{Bu}$
1-50	0. 2813	0. 2282	0. 4905
Theoretical mean	0. 3000	0. 2000	0. 5000

Var ($\hat{n}_D$) = 0. 0003200 Standard deviation = 0. 0179

Var ($\hat{n}_B$) = 0. 0008314 Standard deviation = 0. 0288

Var ($\hat{n}_{Bu}$) = 0. 0002514 Standard deviation = 0. 0159

We have the identity:

Mean of true l_1 for the whole population $= \bar{l}_1$

$$= n_D \bar{l}_{1B} + n_B \bar{l}_{1B} + n_{Bu} \bar{l}_{1Bu} = n_D \bar{l}^*_{1D} + n_B \bar{l}^*_{1B} + n_{Bu} \bar{l}^*_{1Bu} + \Delta$$

If the sample is great we can determine Δ by the following iterative process.

We start by putting $\Delta = 0$ and determine $\hat{n}_D$, $\hat{n}_B$, and $\hat{n}_{Bu}$ under this assumption, say ${}_1\hat{n}_D$, ${}_1\hat{n}_B$, ${}_1\hat{n}_{Bu}$. A new estimate Δ_1 is then found from

$$\text{Mean } l_1 \text{ of sample} = {}_1\hat{n}_D \bar{l}^*_{1D} + {}_1\hat{n}_B \bar{l}^*_{1B} + {}_1\hat{n}_{Bu} \bar{l}^*_{1Bu} + \Delta_1$$

New splitting lines can now be determined and new estimates of n_D, n_B, and n_{Bu}, say ${}_2\hat{n}_D$, ${}_2\hat{n}_B$, and ${}_2\hat{n}_{Bu}$, are found. A new estimate of Δ, say Δ_2, is obtained from:

$$\text{Mean } l_1 \text{ of sample} = {}_2\hat{n}_D \bar{l}^*_{1D} + {}_2\hat{n}_B \bar{l}^*_{1B} + {}_2\hat{n}_{Bu} \bar{l}^*_{1Bu} + \Delta_2 \quad \text{etc.}$$

The process stops when $\Delta_2 = \Delta_{n+1}$.

In this iteration we suppose that the variance of l_1 within a group is known, but if the means of l_1 are determined from older fish we could well expect the variance within a group to be biased too. If this is so we are not able to determine the true discriminant functions. But we can extend the iterative process by using the identity:

Var l_1 for the whole population

$$= \text{Var } l_1 + n_D (\bar{l}_1 - \bar{l}_{1D})^2 + n_B (\bar{l}_1 - \bar{l}_{1B})^2 + n_{Bu} (\bar{l}_1 - \bar{l}_{1Bu})^2$$

$$= \text{Var } l_1 + n_D (\bar{l}_1 - \bar{l}^*_{1D} - \Delta)^2 + n_B (\bar{l}_1 - \bar{l}^*_{1B} - \Delta)^2 + n_{Bu} (\bar{l}_1 - \bar{l}^*_{1Bu} - \Delta)^2 . \tag{1}$$

We start by taking $\Delta_o = 0$ and ${}_o\text{Var } l_1 = a$, where a is chosen at will. Then discriminant functions, classification probabilities and splitting lines are calculated, given first estimates of n_D, n_B, and n_{Bu}, say ${}_1\hat{n}_D$, ${}_1\hat{n}_B$, and ${}_1\hat{n}_{Bu}$. Δ_1 is determined as before and a new estimate ${}_1\text{Var } l_1$ of Var l_1 is determined from (1), replacing Var l_1 by ${}_1\text{Var } l_1$, n_D by ${}_1\hat{n}_D$, etc. and Δ by Δ_1. We can now find new discriminant functions, classification probabilities and splitting lines and also new estimates of the n's, giving a new Δ and Var l_1 etc. The process stops when $\Delta_n = \Delta_{n+1}$ and ${}_n\text{Var } l_1 = {}_{n+1}\text{Var } l_1$.

The iterative process indicated here has not been investigated in detail, owing to a lack of calculating facilities, and should be used with great care. It should in my opinion only be used when the following assumptions are fulfilled:

1. the distributions of l_1, K_2 and VS are normal, with identical variances;
2. the sample size is very great.

As an example of the method the calculations for the 1957 year-class in the spring survey of 1960 are presented below.

The classification probabilities derived from use of the l_1/VS/K_2 discrimination are as follows:

$P_{D,D} = 0.692437$ $\quad P_{B,D} = 0.307511$ $\quad P_{Bu,D} = 0.045310$

$P_{D,B} = 0.250535$ $\quad P_{B,B} = 0.427498$ $\quad P_{Bu,B} = 0.219711$

$P_{D,Bu} = 0.057028$ $\quad P_{B,Bu} = 0.264991$ $\quad P_{Bu,Bu} = 0.734979$

from which we obtain (see App. 4, Equations (12), (13), (14)):

$D = 0.126382 \qquad D^2 = 0.0159724$

$$D_D = 0.207787\, n^*_D - 0.262201\, n^*_B + 0.048194$$

$$D_B = -0.030824\, n^*_D + 0.647127\, n^*_B - 0.140784$$

$$D_{Bu} = -0.176963\, n^*_D - 0.384926\, n^*_B + 0.218973$$

For K_2/VS the classification probabilities are:

$P_{D,D} = 0.529158$ $\quad P_{B,D} = 0.387174$ $\quad P_{Bu,D} = 0.250525$

$P_{D,B} = 0.195648$ $\quad P_{B,B} = 0.241059$ $\quad P_{Bu,B} = 0.181370$

$P_{D,Bu} = 0.275194$ $\quad P_{B,Bu} = 0.371767$ $\quad P_{Bu,Bu} = 0.568105$

and $\quad D = 0.0146802 \qquad D^2 = 0.000215508$

$$D_D = 0.059689\, n^*_D - 0.136649\, n^*_B + 0.009830$$

$$D_B = -0.014278\, n^*_D + 0.278633\, n^*_B - 0.046959$$

$$D_{Bu} = -0.045411\, n^*_D - 0.141984\, n^*_B + 0.051808$$

To get an impression of the variability of the two sets of estimates the variances of the $\hat{n}$'s have been calculated, supposing that $n_D = n_B = n_{Bu}$ and $N = 500$. The results for $l_1/VS/K_2$ and VS and K_2 are presented below:

$l_1/VS/K_2$		VS/K_2	
$\bar{n}^*_D$	= 0.348419	$\bar{n}^*_D$	= 0.388952
$\bar{n}^*_B$	= 0.299248	$\bar{n}^*_B$	= 0.206026
$\bar{n}^*_{Bu}$	= 0.352333	$\bar{n}^*_{Bu}$	= 0.405022

Variances and covariances

		n^*_D	n^*_B	n^*_{Bu}
$l_1/VS/K_2$	n^*_D	0.0004540	-0.0002085	-0.0002455
	n^*_B		0.0004194	-0.0002109
	n^*_{Bu}			0.0004564
VS/K_2	n^*_D	0.0004753	-0.001603	-0.0003151
	n^*_B		0.0003272	-0.0001669
	n^*_{Bu}			0.0004820

The variances of the unbiased estimates $\hat{n}_D$, $\hat{n}_B$, $\hat{n}_{Bu}$ of n_D, n_B, n_{Bu} are

$$\text{Var}\,\hat{n}_D = \frac{\text{Var}\,D_D}{D^2} \quad \text{etc.} \quad \text{(Appendix 4, page 15).}$$

	$l_1/VS/K_2$	VS/K_2
$V(\hat{n}_D)$	$\frac{V(D_D)}{0.126382^2} = 0.004455$	$\frac{V(D_D)}{0.0146802^2} = 0.04834$
$V(\hat{n}_B)$	$\frac{V(D_B)}{0.126382^2} = 0.011544$	$\frac{V(D_B)}{0.0146802^2} = 0.12424$
$V(\hat{n}_{Bu})$	$\frac{V(D_{Bu})}{0.126382^2} = 0.003002$	$\frac{V(D_{Bu})}{0.0146802^2} = 0.02556$

The estimates of variance based on VS/K_2 are ten times greater than those based on $l_1/VS/K_2$.

We may now examine the results of the discriminant analysis in detail.

Spring 1960

App. 4, Table 6 Proportions of the stocks as allocated

	$l_1/VS/K_2$			VS/K_2		
	W	NE	SE	W	NE	SE
n^*_D	0.39	0.32	0.63	0.40	0.31	0.44
n^*_B	0.26	0.27	0.24	0.20	0.20	0.24
n^*_{Bu}	0.35	0.41	0.13	0.40	0.49	0.32
No. of fish	399	500	485	399	500	485

App. 4, Table 7 Unbiased estimates of the proportions of the three stocks

	1_1/VS/K_2			VS/K_2		
	W	NE	SE	W	NE	SE
$\hat{n}_D$	0.48	0.35	0.90	0.44	0.07	0.27
$\hat{n}_B$	0.12	0.19	-0.01	0.17	0.26	0.87
$\hat{n}_{Bu}$	0.40	0.46	0.11	0.39	0.67	-0.14

From the method outlined earlier the variances and covariances of the proportions of the stocks as allocated, n*, have been calculated (App. 4, Table 8).

App. 4, Table 8 Estimates of variances and covariances of the n*'s

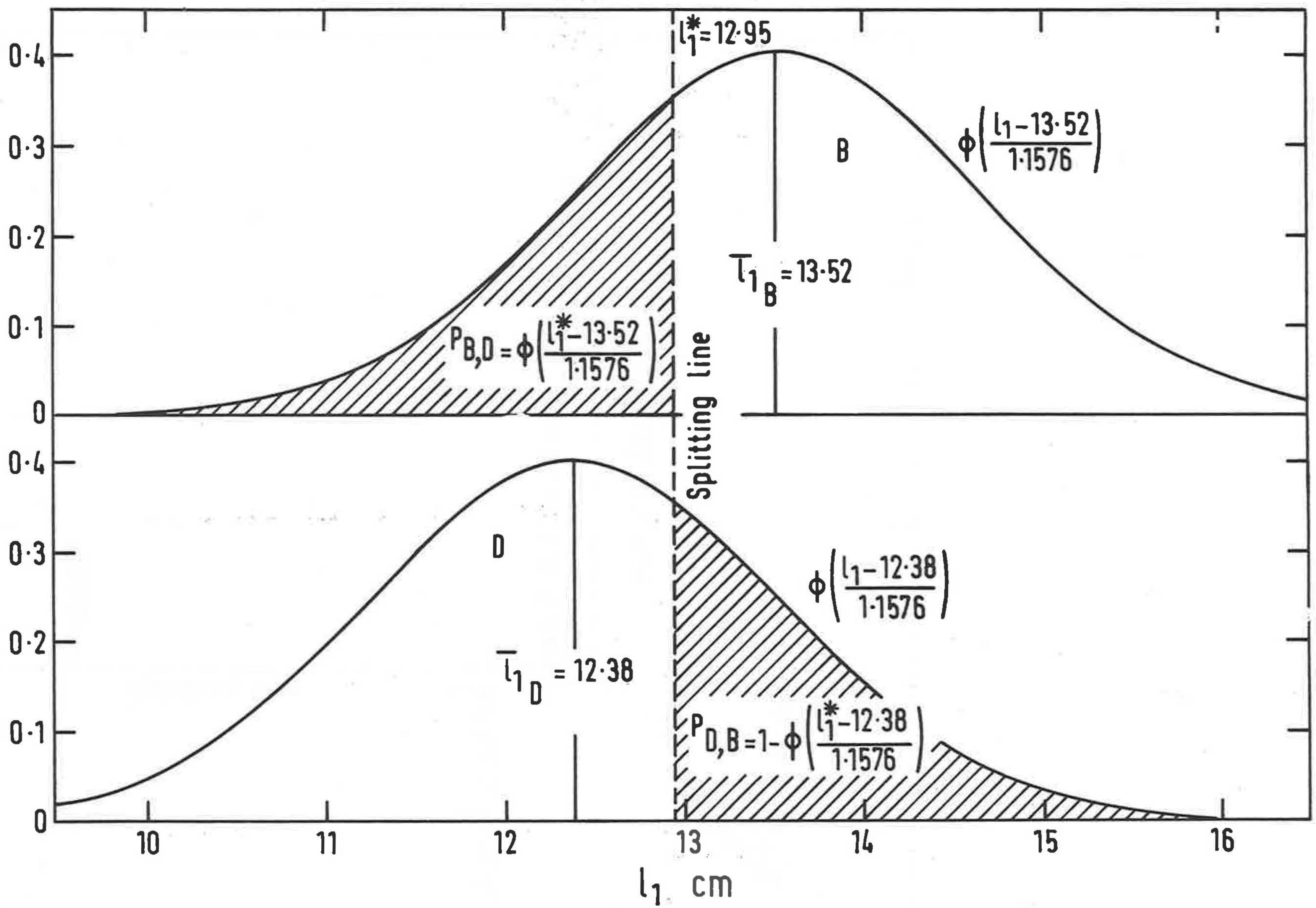
		1_1/VS/K_2			VS/K_2		
		n^*_D	n^*_B	n^*_{Bu}	n^*_D	n^*_B	n^*_{Bu}
W	n^*_D	0.00060	-0.00025	-0.00034	0.00060	-0.00020	-0.00040
	n^*_B		0.00048	-0.00023		0.00040	-0.00020
	n^*_{Bu}			0.00057			0.00060
NE	n^*_D	0.00044	-0.00017	-0.00027	0.00043	-0.00012	-0.00030
	n^*_B		0.00039	-0.00022		0.00032	-0.00020
	n^*_{Bu}			0.00048			0.00050
SE	n^*_D	0.00048	-0.00031	-0.00017	0.00051	-0.00022	-0.00029
	n^*_B		0.00038	-0.00006		0.00038	-0.00016
	n^*_{Bu}			0.00023			0.00045

Finally estimates of the relevant variances and covariances of the unbiased estimates, $\hat{n}$, have been derived.

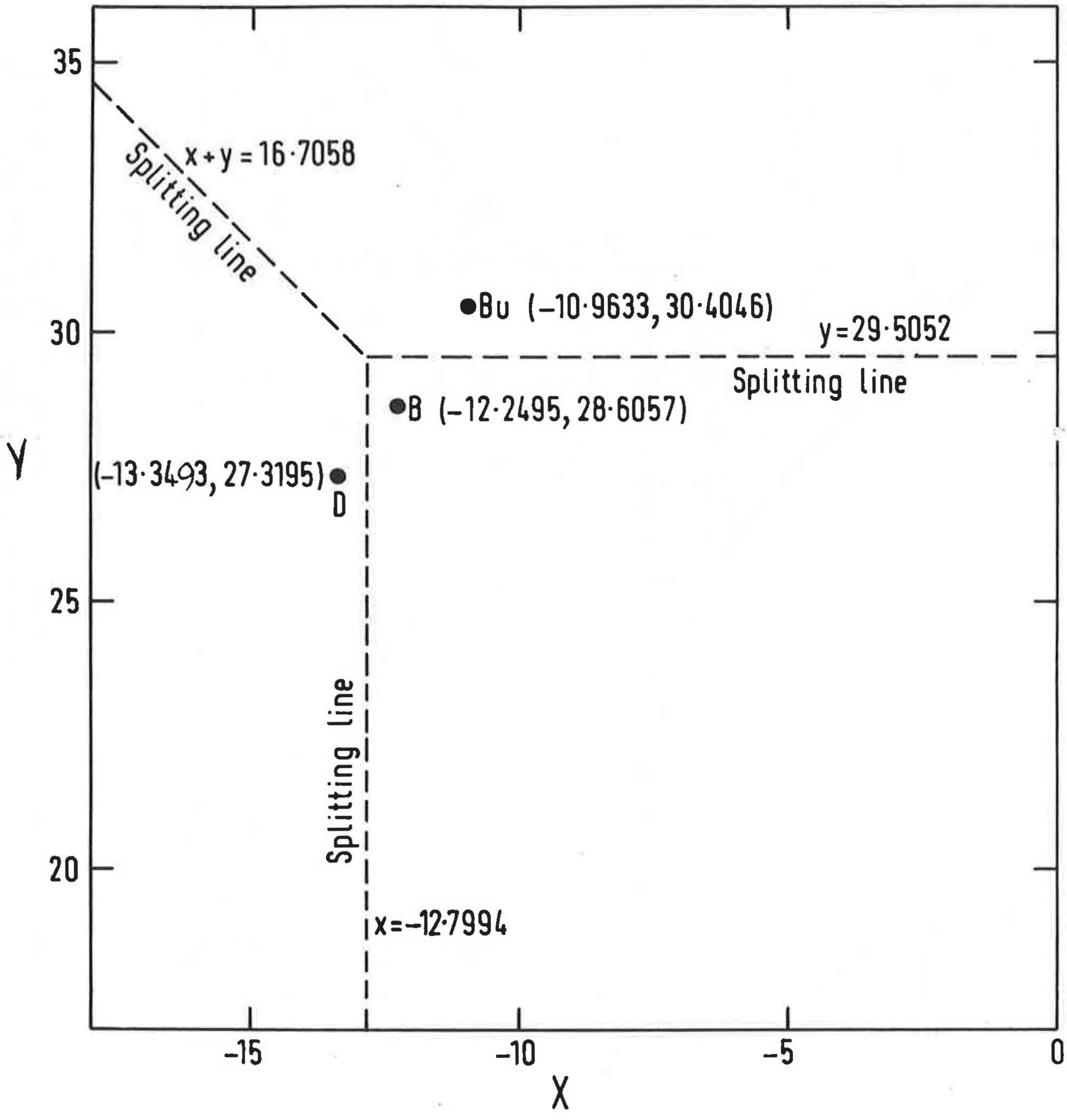
App. 4, Table 9 Estimates of variances, covariances and standard deviations of the $\hat{n}$'s

	$l_1/VS/K_2$			VS/K_2		
	W	NE	SE	W	NE	SE
Var $\hat{n}_D$	0.0054	0.0040	0.0050	0.0597	0.0439	0.0580
s.d. $\hat{n}_D$	0.073	0.063	0.071	0.244	0.210	0.241
Var $\hat{n}_B$	0.0132	0.0107	0.0108	0.1520	0.1201	0.1455
s.d. $\hat{n}_B$	0.115	0.103	0.104	0.390	0.347	0.381
Var $\hat{n}_{Bu}$	0.0035	0.0030	0.0018	0.0312	0.0269	0.0273
s.d. $\hat{n}_{Bu}$	0.059	0.055	0.043	0.177	0.164	0.165
Cov $(\hat{n}_D\ \hat{n}_B)$	-0.0076	-0.0058	-0.0070	-0.0903	-0.0686	-0.0881

It will be noticed that in App. 4, Table 7 negative values are given for the Bank and Buchan stocks in the South-east region. From inspection of App. 4, Table 9 it is seen that the standard deviation for Bank fish, using $l_1/VS/K_2$, in the South-east region is 0.104 and that for Buchan fish, using VS/K_2, is 0.165. The negative values for $\hat{n}_B$ and $\hat{n}_{Bu}$ do not exceed their relevant standard deviations and so may be taken as zero.



App. 4, Figure 1 Theoretical l_1 distributions of Bank, B, and Downs, D, herring, indicating the areas of misclassification of Downs as Bank and Bank as Downs.



App. 4, Figure 2 Positions of Pure Stock centres of the discriminant functions for the Bank, Downs and Buchan stocks, and the relative positions of the splitting lines (for explanation, see text, p. 6).