ICES CM 2012/F

Evaluating how precision in estimates of abundance indices by age from a fisheries-independent trawl survey affects the assessment of Northeast Arctic Cod

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Abstract

The rules for setting quotas for North-East Arctic cod (NEAc) are based on how estimates of stock parameters relate to defined biological reference points obtained from the Extended Survivors Analysis (XSA) model, calibrated using bottom trawl survey data from the Barents Sea. In this paper we use simulations to investigate how the precision in estimates of relevant stock parameters for NEAc relates to different levels of sampling effort in the trawl survey. We also evaluate the importance of estimates of abundance-indices by age as compared to estimates of catch-at-age for assessments and management advice. It is generally assumed that catch-at-age is known exactly and that uncertainty in estimates of abundance is chiefly caused by errors in the survey indices. However, catch-at-age is estimated, and subject to sampling errors that depend on the design and sampling effort in fisheries-dependent surveys. This must be taken into account when evaluating the performance of fisheries-independent surveys. The yearly winter survey used for tuning is expensive, has large area coverage, and samples from 176-394 trawl stations. It is therefore important to establish the required survey effort to achieve adequate precision in estimates of stock parameters. We explore whether the effective sample size for estimating simple statistics, such as the proportion of ages 7+, or mean age, can serve as a proxy. We adopt a statistical catch-at-age model in AD Model Builder which also allows errors in catch at age when evaluating effects

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August 17, 2012

on sampling strategies in the trawl survey on assessment and management advice.

Keywords: Survey, effort, effective sample size, catch-at-age, catch errors, reference points

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The standard assessment of Northeast Arctic (NEA) Cod is based on estimates of spawning stock biomass (SSB) and fishing mortality (F) using the Extended Survivors Analysis (XSA) model, Shepherd (1999). Indices of abundance from yearly scientific bottom trawl and acoustic surveys supply important (and expensive) time-series for calibration (tuning) the model. An important tuning series is based on a stratified, probability-based survey conducted during winter (February-March) to provide abundance indices for NEA cod and other commercially important demersal species. Two Norwegian vessels and one Russian vessel are normally employed in the winter survey, and data on fish abundance are collected using bottom trawls, pelagic trawls, and echosounders (Jakobsen et al., 1997, Pennington and Helle, 2011). The area covered, and the number of trawl stations, have varied from 260×10^3 km^2 and 176 trawl stations to more than 690×10^3 km^2 and 394 trawl stations since 1993.

For the XSA-based assessment it is assumed that the uncertainty in stock assessments based on virtual population analyses (VPA) of catch-at-age data is chiefly caused by errors in the tuning series. A standard assumption in the XSA model is that total catch and catch-at-age can be treated as exact in the VPA calibration. However, catch at age is estimated through expensive fishery catch sampling, and the sampling schemes are constrained by the cost or logistic features of the sampling. Sampling of commercial catches at sea in Norway is conducted by inspectors from the Directorate of Fisheries on-board selected vessels and trips, by the Coast Guard, and through the Reference Fleet managed by the Institute of Marine Research (IMR). The IMR also samples commercial catches at landing ports north of 62°N, with focus on the sampling of cod and haddock for estimating catch-at-age. The sampling of commercial catches often involves stratification and sampling in multiple stages, starting with a list of vessels, ports, or markets. Age-samples are often collected in the last stage, for example by sub-sampling the catch from a fishing trip. This multi-stage sampling requires that estimators of key parameters for total catches in a fishery appropriately accounts for the hierarchical structure of the sample data, and in particular clustering effects that may drastically reduce the effective sample sizes of fish/age-structures for estimating catch-at-age. Assumptions of simple random samples of fish from the total population of fish in the catches cannot be reasonably met when sampling is done in multiple stages. It is therefore prudent that both data and model uncertainties have to be quantified and taken into account when assessing the state of stocks, and in model-based predictions, e.g., of key biological parameters, which are relevant to management decisions. The estimation of catch-at-age of NEA cod with measures of variability is based on a Bayesian hierarchical model Hirst et al. (2005). Because of sampling variability and aging errors, both the survey indices and the VPA estimates at age are rather imprecise (Aanes and Pennington, 2003, Pennington et al., 2002).

In this paper we employ a statistical assessment model to investigate how errors in tuning series and sampling errors in catch-at-age for Northeast Atlantic Cod propagate to the estimates of biological reference points used for quota setting. Given the yearly uncertainty in estimated catch-at-age, we explore how the precision in the reference points for stock assessment of NEA cod change with varying sampling effort for estimating the abundance indices by age used in tuning. Because the precision in abundance indices by age depends on the number of trawl stations and the survey design, we provide estimates of relative standard error in the spawning stock biomass (SSB) for a given effective sample size for estimating the tuning indices. The modeling framework for quantifying reference points and uncertainty is implemented on the Automatic Differentiation (AD) Model Builder Fournier et al. (2011) platform. The reader is referred to Griewank et al. (1991) for a general discussion on AD.

1. The Statistical Models

We adopt the following nomenclature in defining the structural model and the pseudo-observation models:

$$
\begin{aligned}
m &= \text{Number of cohorts},\\
N_{i,j} &= \text{Number of individuals in cohort } i \text{ at age } j,\\
C_{i,j} &= \text{Catch data for cohort } i \text{ at age } j,\\
I_{i,j,k} &= \text{Survey index for cohort } i \text{ at age } j,\\
M &= \text{Natural mortality},\\
q_{j,k} &= \text{Catchability of fish of age } j \text{ belonging to survey } k,\\
\sigma &= \text{Standard deviation for statistical assumption about } I.
\end{aligned}
$$

1.1. The Pseudo-observation Models

Survey Indices

We follow e.g., Patterson (1998) in assuming that individual survey abundance indices are independent, age-structured indices of abundance with log-normally distributed errors, according to (1). In (1), $\mathcal{I}$ is the set of indices $\{i, j, k\}$, such that $N_{i,j}$ is part of the model.

$$\log(I_{i,j,k}) \quad \sim \quad \mathrm{N}(\log(q_{j,k} N_{i,j}), \sigma^2), \forall \{i, j, k\} \ \in \mathcal{I}. \tag{1}$$

Catch Data

Let the true (but unobserved) catch and the observed catches be defined respectively, by $C_{i,j}$ and $\hat{C}_{i,j}$. We assume that $\hat{C}_{i,j}$ is log-normally distributed, i.e., $\log(\hat{C}_{i,j})$ is normally distributed with standard deviation σ_C. In order to relate the true, to the observed catch, we introduce a random variable $u_{i,j} \sim \mathrm{N}(0, 1)$ according to (2). The unobserved catch is now a random (latent) variable with expectation and variance given by (3).

$$C_{i,j} = \hat{C}_{i,j} e^{[u_{i,j}\sigma_C - \frac{1}{2}\sigma_C^2]}, \tag{2}$$

$$\mathbb{E}(C_{i,j}) = \hat{C}_{i,j}, \quad \mathrm{Var}(\log(C_{i,j})) = \sigma_C^2. \tag{3}$$

1.2. The Structural Model

The basic mechanism of the structural model is defined such that each year, the previously appraised stock size, less the mortality due to fishing (catch), is subjected to natural mortality to obtain the current estimate of stock size. Then considering a single cohort (for simplicity and thus omitting the index i) the number of individuals at age j is given by the recurrence relationship (4]). This implementation is analogous to the conventional VPA-XSA approach if we assume the catch data is observed without errors, i.e., $\sigma_C = 0$ in (2).

$$N_j = \Big(N_{j-1} - C_{j-1}\Big) e^{-M}. \tag{4}$$

The statistical assumptions in (1) leads to a log-likelihood response function defined by (5), where the $N_{i,j}$ which are not estimated directly, are derived using the recurrence relation in (4).

$$l(N, q, \sigma, M) = \sum_{\{i,j,k\} \in \mathcal{I}} \left[-\frac{1}{2\sigma^2} \left\{ \log \left[\frac{I_{i,j,k}}{(q_{j,k} N_{i,j})} \right] \right\}^2 - \log(\sigma) \right]. \tag{5}$$

We account for uncertainty in the catch data by transforming the recurrence relation (4) into (6).

$$N_j = \left(N_{j-1} - \hat{C}_{j-1} \cdot e^{\left[u_{j-1}\sigma_C - \frac{1}{2}\sigma_C^2\right]}\right) e^{-M}. \tag{6}$$

We have further assumed that the for each survey indexed by k, catchability is age (j) dependent, and described by a sigmoid function given by (7), where q_0, α and β are parameters to be estimated.

$$q_{j,k}|(q_{0,k}, \alpha_k, \beta_k) = q_{0,k} \frac{e^{(\alpha_k j + \beta_k)}}{1 + e^{(\alpha_k j + \beta_k)}} \tag{7}$$

Considering a single survey (dropping the index k), it can be easily shown that the log-likelihood function for this model is defined by (8).

$$\hat{l}(N, q_0, \alpha, \beta, \sigma, M, \sigma_C, u) = l(N, q_0, \alpha, \beta, \sigma, M) + \frac{1}{2}\|u\|_2^2, \tag{8}$$

where $l(N, q, \sigma, M)$ is the likelihood function given in equation (5), but with the underlying recurrence relation (4).

For computational ease, we optimize the marginal log-likelihood function defined by (9), where the latent variables are removed by integration.

$$\log \int \exp\left(\hat{l}(N, q_0, \alpha, \beta, \sigma, M, \sigma_C, u)\right) \quad du, \tag{9}$$

This high-dimensional integral, which can be computationally intractable, is computed using a Laplace approximation; see (Skaug and Fournier, 2006, Tierney and Kadane, 1986). The resulting objective function approximation based on the Laplace approximation is defined by (10).

$$\tilde{l}(N, q_0, \alpha, \beta, \sigma, M, \sigma_C) = -\frac{1}{2}\log|H(\hat{u})| + \hat{l}(N, q_0, \alpha, \beta, \sigma, M, \sigma_C, \hat{u}), \tag{10}$$

$$\hat{u} = \text{argmax} \quad \hat{l}(u|N, q_0, \alpha, \beta, \sigma, M, \sigma_C), \tag{11}$$

and $H(\hat{u})$ is the Hessian of $\hat{l}(u|N, q_0, \alpha, \beta, \sigma, M, \sigma_C)$, evaluated at $\hat{u}$. Thus for each choice of the variables $(N, q_0, \alpha, \beta, \sigma, M, \sigma_C)$, an inner optimization problem where only u is variable, is solved to determine $\hat{u}$ and $H(\hat{u})$. The computational framework is available on the ADMB (ADMB Project 2009, Fournier et al. (2011)) platform.

2. Data Description

The adopted method for estimating the Norwegian catch at age of NEA cod (ICES (2012)) is described in Hirst et al. (2005). This is a Bayesian approach and includes the posterior distribution of catch at age which represents the sampling error. The Norwegian catch at age comprises approximately half of the total catch, and the uncertainty in the remaining catch is not reported. The annual sampling distribution of the Norwegian catches can adequately be approximated by a multivariate log normal distribution, i.e. $\log(\hat{C}_{.y})\ N(\log(C_{.y}), \Sigma_y)$ where $\log(C_{.y})$ is the mean vector of log catch in year $y(\equiv i+j)$ and Σ_y its covariance matrix with diagonal elements $\sigma^2_{j,y}$. The relative standard error of $\hat{C}_{j,y}$, given by (12), is independent of the mean (constant CV).

$$\mathrm{RSE}(\hat{C}_{j,y}) \quad = \quad \frac{\mathrm{SE}(\hat{C}_{j,y})}{\mathrm{E}(\hat{C}_{j,y})} = \sqrt{\exp(\sigma^2_{j,y}) - 1}. \tag{12}$$

We assume that the error in the additional catch, and covariance structure, is the same as in the Norwegian catch and that the point estimates of the total international reported catch at age are unbiased. More specifically, we use the available point estimates for catch at age reported by AFWG (ICES (2009), Table 3.7) adding number of cod consumed by cod (ICES (2009), Table 3.8) to use the same data used for the final VPA run (Figure 1). Then we adopt Σ_y as estimated from the Norwegian catches to represent the error in the total catch at age used for input to final VPA. The error structure for the catch is shown in Figure 2.

The survey indices considered is from the Norwegian bottom trawl survey (Jakobsen et al. (1997)), which is a stratified systematic survey for 1996 through 2008. We have followed the approach in Jakobsen et al. (1997) to estimate the abundance index at age but estimated the stratified mean abundance per km^2 rather than the swept area estimates used in routine assessments (e.g. ICES (2012)) to reduce potential effect of year-to-year variation in the surveyed area. The survey indices are shown in Figure 1. To estimate the precision in the survey we assumed stratified simple random sampling and resampled stations with replacement independently within stratum. This is a standard bootstrap approach and a large number of replicates estimates the sampling variability in the survey. The relative standard error by age and year is shown in Figure 2.

The number of stations sampled in the survey varies from 176 to 394 with a median number of 262. The effect varying sampling intensity on the survey was assessed by varying the total number of stations sampled from each trip in the resampling procedure. In this way we generated time series of distributions of survey indices corresponding to varying sampling intensity from 50 trawl stations to 500 trawl stations each year. The numbers of trawl stations was allocated to strata adopting the same proportions of stations to each stratum as in the original survey. The effect of varying sampling intensity on the precision as a function of abundance is shown in Figure 3.

3. Analytical Evaluation of Design Efficiency

A sampling unit selected in the first stage in multi-stage sampling is called a primary sampling unit (PSU). In trawl surveys, the trawls stations form the primary sampling units, and sampling for age and length is therefore multi-stage sampling.

In general for biological samples, the PSUs will contain a group or cluster of individuals. Examples are all the fish caught by a survey trawl haul (the PSU) or the fish sampled from a fishing trip (the PSU). Because fish that are caught together tend to be more similar than the fish in the entire target population (i.e. there is positive intra-cluster correlation), the effective sample size will be much smaller than the total number of fish sampled (Bogstad et al., 1995, Pennington et al., 2002, Pennington and Helle, 2011, Pennington and Volstad, 1994).

The efficiency of each survey design is evaluated by comparing the respective design-based variance of the estimated survey index (I) with the expected variance obtained under simple random sampling. (Kish, 1965, 1995) defined the design effect, d_{eff} in (13), as the ratio of the two variances.

$$d_{\text{eff}} \quad = \quad \sigma_c^2(\bar{I}_c)/\sigma_{\text{srs}}^2(\bar{I}_{\text{srs}}), \tag{13}$$

where $\sigma_c^2(\bar{I}_c)$ is the variance based on the actual (complex) survey design, and $\sigma_{\text{srs}}^2(\bar{I}_{\text{srs}})$ is the expected variance under simple random sampling (SRS) for a sample of equal size. The design-based variance, $\sigma_c^2(\bar{I}_c)$, reflects the effects of stratification and, for the transect survey, clustering of stations. Kish (1995) and (Potthoff et al., 1992), provide a general discussion on the calculation of design effects and effective sample sizes while (Cochran, 2007) p.136 gives an estimator for $\sigma_{\text{srs}}^2(\bar{I}_{\text{srs}})$ The effective sample size for estimation of the mean CPUE ($\bar{I}$) using data from the complex survey design C is defined by (14).

$$n_C^* = n/d_{\text{eff}}. \tag{14}$$

Thus the effective sample size n_C^* is the number of stations selected by simple random sampling that would be required to achieve the same precision obtained with n stations under the actual complex sampling design. If, for example, the design effect equals 0.5 for the estimated abundance indices for a survey with 200 trawl stations in a stratified design, then a simple random sample of 400 stations (the effective sample size) would be required to achieved the same precision.

4. Modeling Approach

We perform $n_r = 100$ replicates of the survey index for six scenarios of survey effort and design, as given by the sample size at the primary level. For the survey trawl data considered in this paper, the effective sample size at the primary level is the number of trawl stations, n. Hence $n \equiv (50100200300500)$. Using the ADMB platform, we determine maximum likelihood values for 26 parameters based on survey indices and catch statistics spanning the period 1996–2008. We next generate maximum likelihood estimates for the ssb and N_{7+} during the same period, based on the optimized parameters.

We monitor the model predictions of two population parameters namely, the spawning stock biomass (ssb) and the size of fish with ages 7 or more (N_{7+}). We investigate whether our modeling framework is capable of capturing the temporal variations in these parameters by plotting the ensemble of our model predictions together with the XSA-VPA predictions. It must be recalled however, that our modeling framework – unlike the XSA-VPA approach – does not assume that the catch data is without error. Thus in principle, we would only expect that our model predictions capture the trend in the ssb and N_{7+}, which are fundamental to the stock dynamics.

For each n and year, we calculate the ensemble mean (μ), standard deviation (σ) and the coefficient of relative standard error R$= \frac{\sigma}{\mu}$. These derived parameters are useful in addressing questions of (i) survey precision and (ii) optimal n, which provides a trade-off between cost and precision. The relative standard error, which is indicative of survey precision, would be expected to decrease with increasing n (the number of trawl stations). We adopt a power-law in parameterizing σ as a function of n, as in (15).

$$\sigma(c, \eta|n) = cn^{\eta}, \quad \eta \leq 0. \tag{15}$$

Given that we are limited to a finite number of the tuple (σ n), the parameterization allows us to make inference about the asymptotic behavior of σ, and also in deducing the optimal number of trawl stations, beyond which there is no significant variance reduction in the estimate of either N_{7+} or ssb.

5. Simulation Results

We performed $n_r = 100$ replicates of the survey index for six values $n \equiv (50\ 100\ 200\ 300\ 500)$. For the sake of brevity, we present results for three sample sizes (50, 300 and 500) in Figure 4, which shows results for the model predictions of N_{7+} and ssb. Figure 4 presents three significant observations:

[a] the ssb and N_{7+} exhibit identical temporal trends,

[b] prediction ssb has consistent trend with the XSA-VPA estimates,

[c] the variance (both for ssb and N_{7+}) decreases with increasing n.

Observation [a] is consistent with the literature (see Pennington et al. (2011)), i.e., that the proportion of N_{7+} can serve as proxy for the ssb. Observation [b] implies that our parsimonious model captures the same population dynamics as the XSA-VPA model. Figure 5b. show the relative standard error per trawl haul calculated for all years. The figure shows that for all years and n, the relative standard error is less than 20%. Though the relative standard error values give indications to precision levels, there is still a need to determine an optimal value for n, which represents a trade-off between data precision and sampling cost (each additional trawl haul translates into added data collection cost). Using observation [c], we parameterized σ for N_{7+}, $\sigma_{N_{7+}}$, as a function of n, and Figure 5a. shows an example for years 1996–1999. Figure 6a. shows the parametric representation of $\sigma_{N_{7+}}$ for all years (1996–2008). We determined the asymptotic value for n, n_∞, beyond which the change in σ is less that a given threshold (here set to 10^{-4}), by differentiating the functional representation of $\sigma_{N_{7+}}$. Figure 5b. results from the change in $\sigma_{N_{7+}}$ with respect to change in n.

Table 1 summarizes the annual number of sampled trawl stations (n), the derived asymptotic number of stations (n_∞) for N_{7+}, the effective sample size for the total density (including all ages) $n_{\text{eff}}^{(\text{tot})}$ and for the density of fish 7 years and older n_{eff}^{7+}. Observe that in general, the effective sample size for the total density (including all ages) $n_{\text{eff}}^{(\text{tot})}$ and for the density of fish 7 years and older n_{eff}^{7+} for each year are both higher than the actual number of stations sampled. Further, considering only the 7+ age group, the number of stations sampled annually is in general, much higher than the asymptotic number of trawl stations required.

6. Conclusions

We have used a simulation based approach to investigate how the precision in ssb and N_{7+} estimates for NEAc relates to different levels of sampling effort in the trawl survey. The modeling approach used a parsimonious model which incorporates uncertainty in the catch data when evaluating model fit to observation data. Our results indicate that in general, the yearly winter survey has precision with relative standard error (R) less than 20%. We have also established that the proportion of fish aged 7+ and the spawning stock biomass are good proxies for evaluating the optimal trade-off between effort and precision. For the particular period covered by the simulations, our results indicate that asymptotic values for the number of trawl stations to be sampled range between 160–273, with an average value of $n \approx 208$ and a standard deviation of about 37 trawl stations.

Considering the estimates of effective sample size for the total density (including all ages), and of fish 7 years and older, our analysis indicates that the survey is relatively efficient for these parameters.

Acknowledgement

This work has been funded by the ADMAR (Adaptive management of living marine resources by integrating different data sources and key ecological processes) project at IMR, Norway.

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7. Tables and Figures

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Year	n	n_∞	$n_{\text{eff}}^{(\text{tot})}$	n_{eff}^{7+}
1996	306	273	318	686
1997	176	274	189	294
1998	200	241	158	237
1999	219	205	479	570
2000	242	160	312	592
2001	348	201	580	796
2002	394	205	846	342
2003	277	171	142	688
2004	270	167	526	479
2005	262	186	427	485
2006	263	191	212	572
2007	257	230	425	570
2008	234	202	243	379

Table 1: Annual number of sampled stations (n), derived asymptotic number of stations (n_∞), estimated effective sample sizes for the estimated total density ($n_{\text{eff}}^{(\text{tot})}$) and for the density of fish age 7 years and older (n_{eff}^{7+}) for the Norwegian bottom trawl survey for NEA cod.

List of Figures

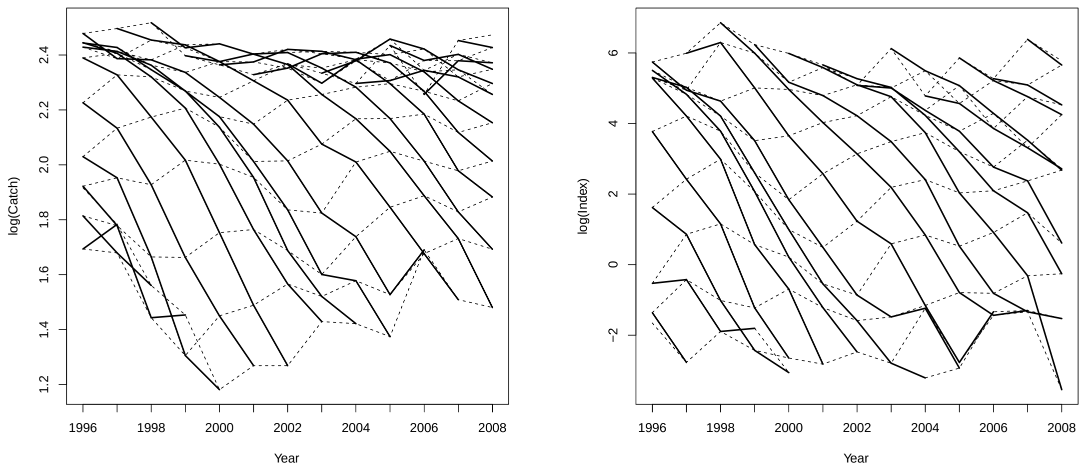


Figure 1: **Left**:Reported log total catch of NEA cod by ages (3-12) and years (1996-2008). The broken lines traces the ages while the solid lines traces the cohort , and **Right**: Estimated log density of NEA cod (log(Index)) by ages (3-12) and years (1996-2008). The broken lines traces the ages while the solid lines traces the cohort.

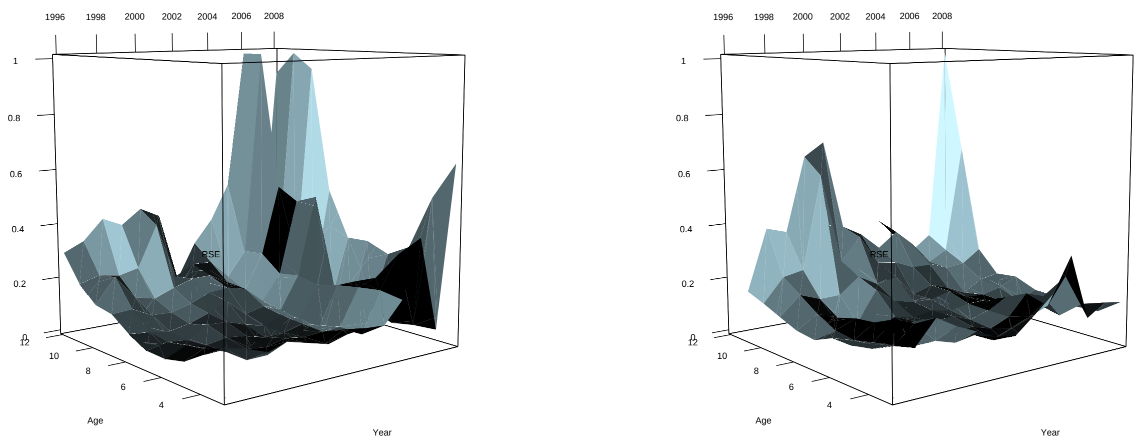


Figure 2: **Left**: Estimated Relative Standard Error (RSE, z-axis)of total catch by age (3-12) and year (1996-2008) , and **Right**: Estimated Relative Standard Error (RSE, z-axis)of estimated indices by age (3-12) and year (1996-2008)

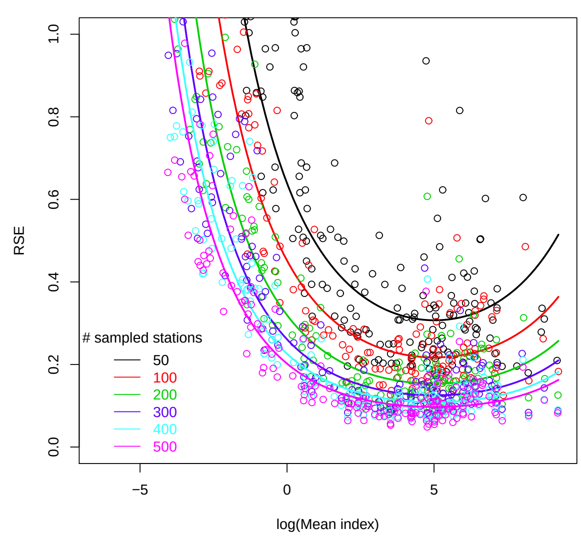
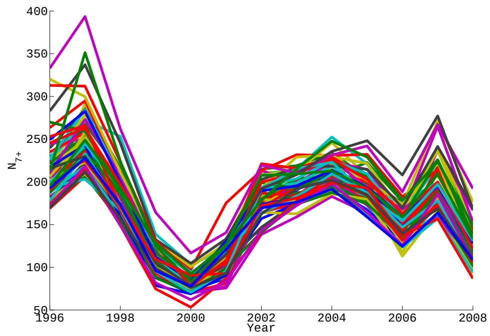
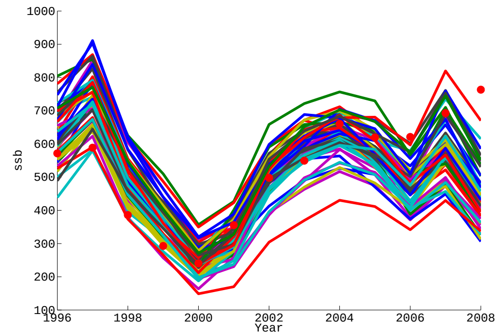
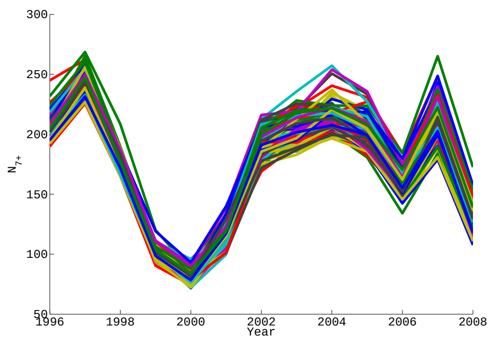


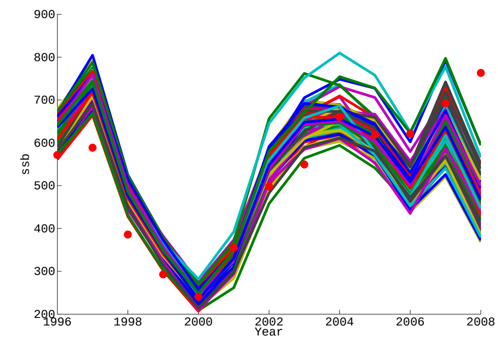
Figure 3: Estimated Relative Standard Error vs log of mean indices by varying sampling intensity indicated by different colors using data for NEA cod ages 3-12 for 1996-2008. The solid lines show the mean RSE for each sampling intensity



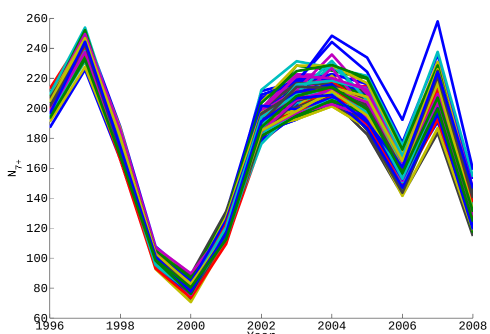


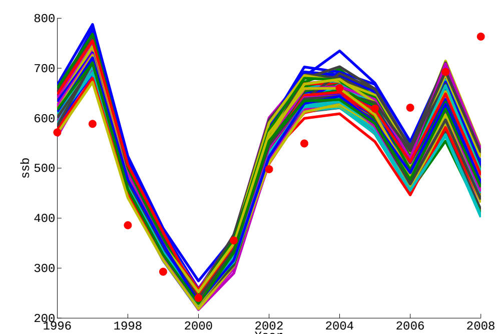
a. N_{7+} for $n = 50$ b. ssb for $n = 50$





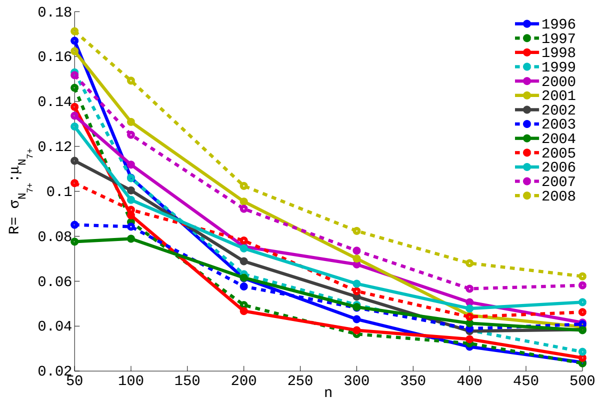
c. N_{7+} for $n = 300$ d. ssb for $n = 300$



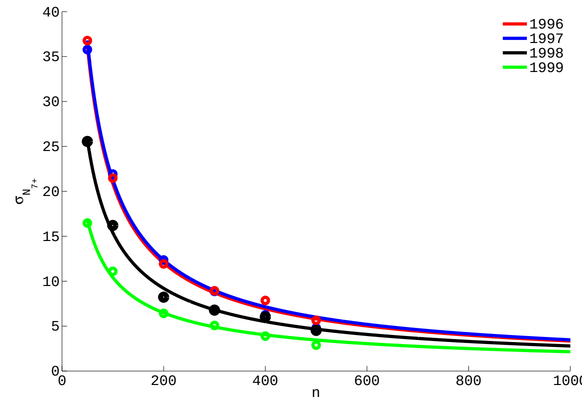


e. N_{7+} for $n = 500$ f. ssb for $n = 500$

Figure 4: **Left**: Stochastic realizations of population size for 7+ group, based on 100 bootstrap survey indices in the period 1996–2008. **Right**: Corresponding estimates of the spawning stock biomass (ssb) and a plot of estimates from the 2008 converged VPA (red dots).

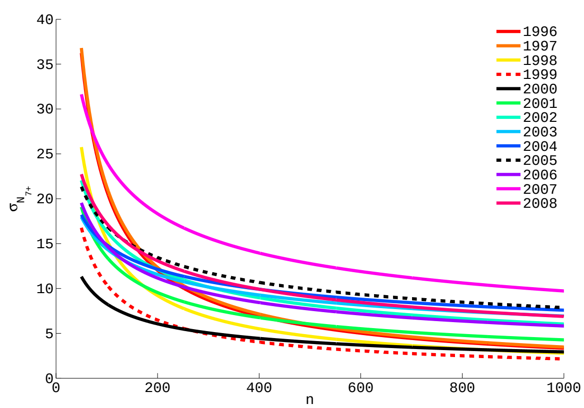


a. Relative standard error (R) for N_{7+}

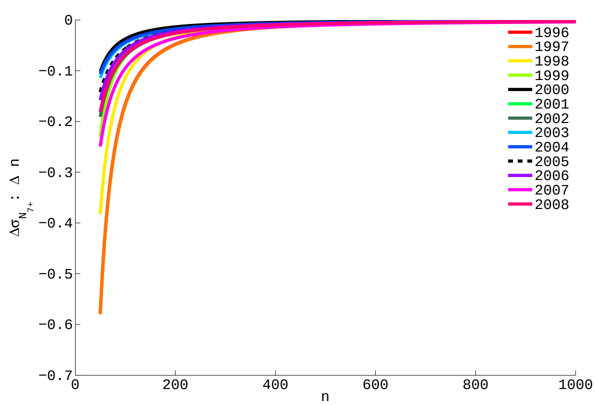


b. $\sigma_{N_{7+}} = cn^{\eta}$.

Figure 5: **Left**: Survey precision (1996–2008) – the relative standard error (R) for N_{7+} estimates. **Right**: Example parameterization of $\sigma_{N_{7+}}$ as a function of n, and extrapolated to $n = 1000$. The red dots represent calculated values based on 100 bootstrap replicates.



a. $\sigma_{N_{7+}}$ vs. n



b. plot of $f = \frac{d\sigma_{N_{7+}}}{dn}$

Figure 6: **Left**: Estimates of $\sigma_{N_{7+}}$ derived from the stochastic realizations are fitted to a power function $\sigma_{N_{7+}} = cn^{\eta}$ and extrapolated to cover $n = 1, \ldots, 1000$, and **Right**: plot of the change in $\sigma_{N_{7+}}$ with respect to n