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Testing various geostatistical models to combine bottom trawl catches and acoustic data

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ABSTRACT

The aim of the CATEFA project is to combine information on demersal fish stock abundance from acoustic and bottom trawl surveys. While acoustic data are collected continuously while the research ship is underway, it is likely that combining two sources of information on the same variable should improve abundance estimation. A variety of geostatistical models are compared, contrasted and the output described. In this study, twenty scientific surveys from three areas (the North Sea, the Irish Sea and the Barents Sea) are analysed. These 3 zones have diverse species assemblages and hydrographic environments. Nevertheless we manage to find models relevant to most of these different situations. Unfortunately, however, we show that this enhancement can increase the variance of the estimation because of the inherently high variability of acoustic recordings.
The purpose of this paper is to compare the results of three geostatistical models (i.e. co-kriging, model with orthogonal residuals, kriging with external drift) in terms of precision, details of maps, variance local and global of the estimation's errors and cross validation.
The role of the acoustic in each model and the precision it brings, is then discussed.

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Introduction

Bottom trawl surveys are commonly used in the assessment of demersal fish abundance. It is now routine to collect simultaneous acoustic survey data while carrying out a bottom trawl survey. The aim of the EU-framework 5 project CATEFA is to evaluate how the acoustic recordings could improve the estimation provided by catches. But what criteria can be used to decide if a model is better than an other? For example, a model combining acoustic and trawl can bring more details than a model using only trawl data, while the variance of the two estimations can be similar. Qualitative comparisons like the pattern of the interpolated map, statistical summaries of the models errors, or confronting the output to the expert judgement are possible solutions.
In a geostatistical context, this paper discusses the results of three different models: two of them using both acoustic and trawl data and the third using only the catches. The aim is to appraise if the acoustic recording bring important improvement in the estimations or if the models with and without acoustic are equivalent to assess fish abundance.

Materials

The data

Eighteen scientific surveys have been used for the exploration of the data: six surveys from the Barents Sea (between 1997 and 2002), three surveys from the Irish Sea (between 2000 and 2002) and nine from the North Sea (three English, two French and four Scottish surveys between 1999 and 2003). Nevertheless, only nine of them corresponding to the assumptions required by the models (see further), were used for the modelisation: four in the Barents Sea, two in the Irish Sea and three in the North Sea.

In the Barents Sea, we have been using the Norwegian demersal surveys combining acoustic and bottom trawl carried out by the Institute of Marine Research (Bergen). Sampling follows a regular grid with a haul every 20 n.mi. (Fig. 1.a) The number of hauls varied between 200 and 300, while between 5000 and 7000 data are collected between the stations. The surveys 1997, 1998, 2000 and 2001 have been used. However, large concentrations of fish tend to occur where the surface temperature is below zero (the polar front effect). Since outliers typically have a large impact on the results, this part of the data needs particular attention. As a first step we have chosen to exclude trawl stations with surface temperature below zero from the analysis.

The Northern Irish Bottom Trawl Surveys carried out by DARDNI (Belfast) are used in the Irish Sea. These surveys are mostly small (20 or 30 hauls and 170-230 recordings between stations). They follow a random sampling design stratified by depth and stratum: sand/gravel (Fig. 1.b). Nine strata have been differentiated in all. The two surveys: winters 2000 and 2002 have been used.

In the North Sea, the surveys rise from the ICES co-ordinated International Bottom Trawl Surveys (IBTS). They follow a random design stratified by ICES rectangle (Fig. 1.c). Trawls and acoustic data are only taken in daylight hours. Each survey comprises between 60 and 80 hauls and between 400 and 1000 acoustic data. Only three FRS's surveys (1999, 2002 and 2003) were able to be used.

In the Irish and the North Sea, because no correlation between acoustic and trawl data could be seen, some very large values have been removed from the datasets for the modelisation (one to five data maximum per survey). Some very large school of fish, about 100 or 1000 times larger than the average of the data, have a random distribution which often hide the behaviour of the lower values.

The acoustic back-scattering energies were converted to Nautical Area Scattering Coefficient - NASC (MacLennan *et al.*, 2002) and expressed in $m^2 \cdot n.mi^{-2}$. The integration threshold was set at -70dB. NASC values were available both during and between trawl stations. For the on station NASC, integration was carried out for the whole trawling period. In general the tows were standardised within each survey series. For each survey series, the NASC values between trawl stations were available at fixed Elementary Sampling Distance Units (ESDU). As the ESDUs were different from tow average lengths, between station NASC values were converted (i.e. regularized) to produce ESDU as close to the tow average lengths as possible for each survey series, namely, 3 n.mi in the Irish Sea, 1 n.mi. in the Barents Sea, and 2 n.mi in the North Sea.
Vertically, the acoustic of the forty first meters above the bottom was integrated in the Barents Sea and the five first meters for the rest of the datasets.

This kind of sampling design, often called "double sampling", in which an auxiliary variable is available on a larger sample than the variable of interest, is detailed in Cochran (1977).

To get variables with comparable units, the fish catches are turned into an equivalent acoustic energy, i.e. the acoustic energy that the fish caught in the trawl should have generated. Because fish characteristics influence this transformation, two groups of fish have been used: demersal (bottom) fish and pelagic (mid water) fish. For each group of fish, the equivalent NASC of the corresponding fish in the net is provided. The trawl variable will refer alternatively to the demersal or the pelagic equivalent NASC depending on which of these two variables happen to get larger correlation with the acoustic variable.

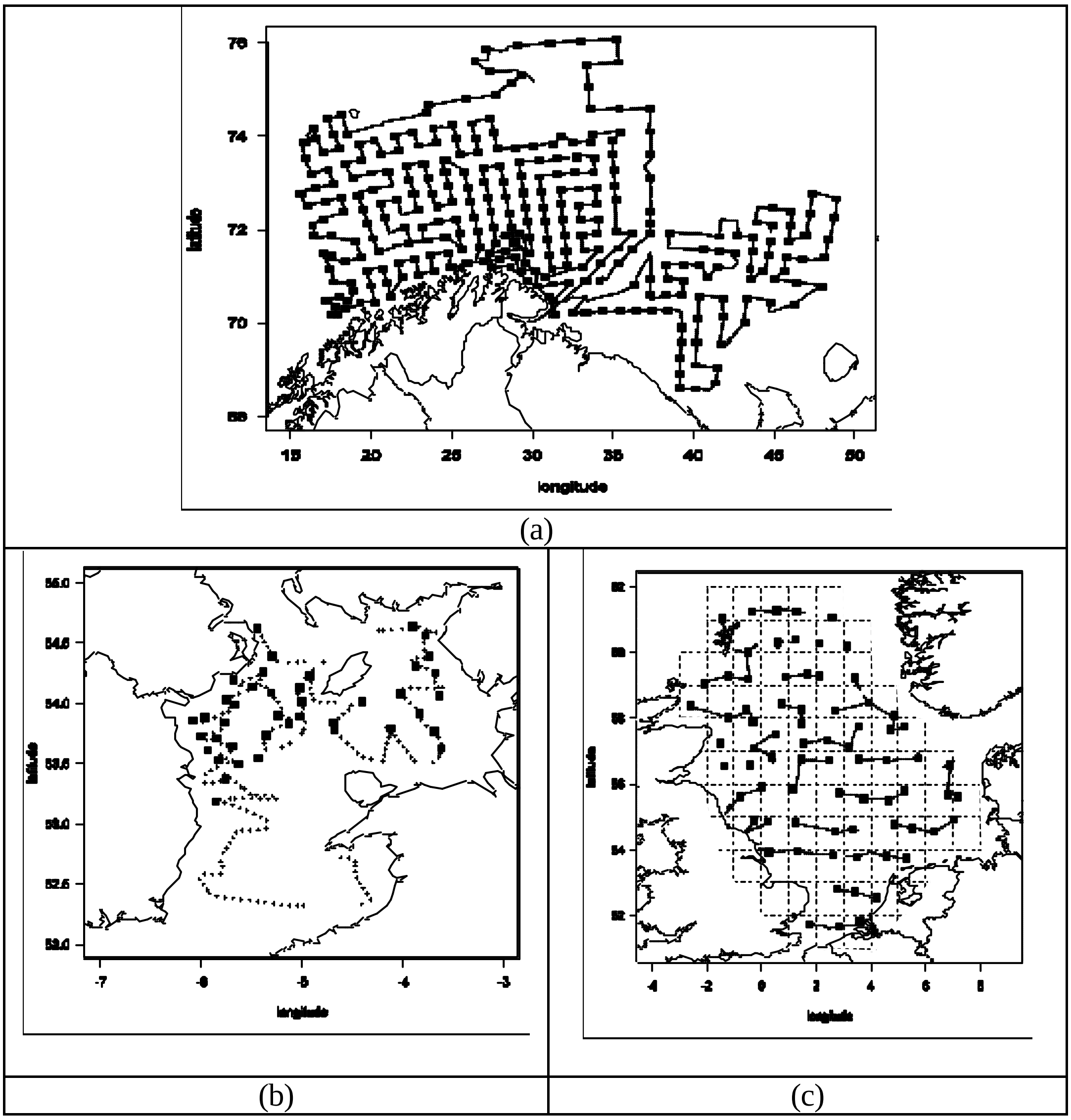

Figure 1. Study areas (a) the combined acoustic and bottom trawl surveys for cod and haddock in the Barents Sea, (b) the Northern Irish Bottom Trawl Surveys, in the Irish Sea and (c) the International Bottom Trawl surveys in North Sea – IBTS. Solid squares represent stations. Lines represent between stations recordings.

Discrepancy between variances

Skew distributions are difficult to sample and, the experimental variance of a given number of samples from a skew distribution may vary considerably around the true value especially when the number of samples is low (low with regards to the actual variance). In skew distributions, few very large but also very rare values, generate high variance. Larger is the number of observations, larger is the probability to meet them.

We observe (Table 1) that the ratio k^2 between the variance of the underway acoustic observations (few thousands data) and that of on station observations (few hundreds data) diverges from 1.

$$k^2 = \frac{\operatorname{var}(underway\ acousticdata)}{\operatorname{var}(onstationacoustic\ data)}$$

To persuade that this phenomenon is general to the "double sampling" of skew distributions and are in no way particular to the data used in this study, we have simulated 500 sets of 7000 lognormal data (independently) from which 500 subsets of 300 points have been taken randomly (7000 corresponds to the number of underway samples and 300 to the number of stations in Norway 2000). The variance and the mean of the simulated lognormal distribution are equal to the mean and the variance of the acoustic underway in 2000 (m = 54 and s^2 = 122826). In 80% cases, the ratio k^2 between the empirical variance of the main 7000 samples and the empirical variance of the 300 subsamples is greater than 1 (Figure 2). The value 1.35 observed in 2000 (represented by a vertical thick line) is among the most possible, it is below the mean simulated ratio (=1.82). When a large value is taken, the variance of the subsample becomes extreme because of the small number of samples.

So the observed discrepancy between the experimental variances can be interpreted as a sampling problem: double sampling or heterotopic sampling in geostatistical terms (the set of station is very small compared to the set of samples of the acoustic variable) of skewness distributions.

With the aim of using the acoustic recordings between stations to improve the catch estimation, this difference of variance between the two level of samples is very central.
First, the high variance of the acoustic underways informs about the variance which could probably be observed if the catch variable was known as well as the acoustic variable. In other words, the underway acoustic variance is larger but also more realistic than the on-station variance.
In addition, the estimation variance is generally larger when the variability of the data increases.

Then, if we want to compare the real performance between a model which only uses an on-station variable and a model which uses underways, it is necessary to rescale the variable to correct this effect. The variables have to be normalized.

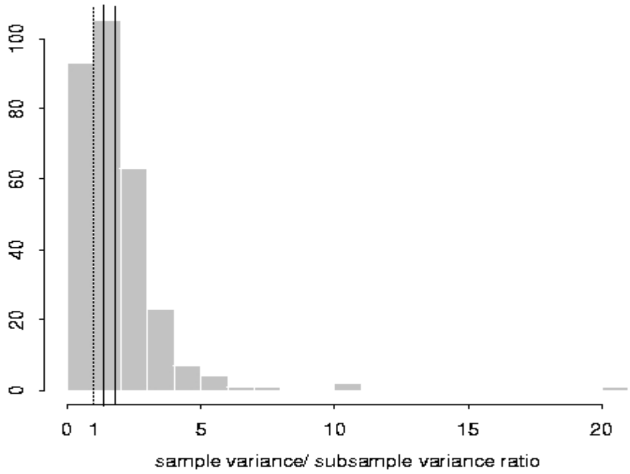

Figure 2. Histogram of ratio between the empirical variances of the main sample (7000 points) and the subsample (300 points) for 500 draws of a lognormal distribution with the mean and the variance of the acoustic for the 2001 survey. The dotted vertical line is equal to 1 and the second one (=1.35) is equal to the observed ratio, and the third one (=1.82) is the average of the distribution.

Table 1 Ratio between the variance of the underway acoustic observations and the variance of the on station acoustic observations

Survey	Year	Number of data underway	Number of data on station	k^2
Norway	1997	5209	176	1.33
	1998	5135	198	1.83
	2000	7680	302	1.35
	2001	7666	300	3.55
Ireland	2000	110	37	0.3
	2002	176	41	0.23
Scotland	1999	468	44	4.6
	2002	430	47	33.0
	2003	303	46	0.7

Methods

The first part of the methods gives details about the models used, and a second part describes tools to compare them. The models theory is not really required for the second part but is present here for a more precise understanding of the approach.

Models description

Three geostatistical models are compared.

Basically, the following random functions are used:
$T(x)$ Trawl variable, i-e demersal or pelagic equivalent NASC. It is the Target variable, the variable to assess.
$T(x_i)$ Trawl variable at sampling locations $x_i \in \{stations\}$
$A(x)$ Acoustic variable, the Auxiliary variable.
$A(x_i)$ Acoustic variable at sampling locations $x_i \in \{stations + underways\}$

- **Ordinary kriging**

In this model, the acoustic is not used. Only the catch data observed at different location are employed in the model.

$$T^K(x_0) = \sum_{\substack{stations \\ \in \\ neighbourhood}} l_i^T T(x_i)$$

Knowing the covariance function of the trawl variable, the kriging weights l_i^T are chosen to insure no bias and to minimize the estimation variance.

This basic model, where acoustic is not taken into account, is presented to be compared to the other models and to evaluate the improvement provides by the addition of the acoustic.

- **Heterotopic Cokriging**

The cokriging estimator is a linear combination of data from different variables located at different samples points, like acoustic and catch (Wackernagel, 1998).

$$T^{CK}(x_0) = \sum_{\substack{stations \\ \in \\ neighbourhood}} l_i^T T(x_i) + \sum_{\substack{stations \\ + underways \\ \in \\ neighbourhood}} l_i^A A(x_i)$$

As for kriging, the cokriging weights l_i^T and l_i^A are chosen to insure no bias and to minimize the estimation variance.
This estimator is interesting for this study because it allow assessing the catch with every sample available. However the spatial structure of each variable and the cross structure are needed.

The heterotopic cokriging is, in fact, difficult to use in practice. Structure and cross structures between the variables are required, which are often unstable because of the skew distributions and the choice of the neighbourhood is also a delicate point. Some simplifications are then welcome (Matheron 1979).

The Markov-type model (or model with orthogonal residual) is a simplification of the classical cokriging (e.g. Rivoirard, 2001). In this model, the catch variable is supposed be factorized with the two orthogonal factors, the acoustic *A(x)* and a residual *R(x)* :

$$\left\| \begin{array}{l} T(x) = a\,A(x) + R(x) \\ \text{with } \operatorname{cov}\left[A(x), R(x+h)\right] = 0 \quad \forall x \forall h \end{array} \right.$$

This assumption is equivalent to verify that the structure of the catch variable is proportional to the cross structure acoustic-catch (Bouleau and Bez, 2004).

Thanks to be spatial independence between A(x) et R(x), the cokriging of the target variable reduces to the sum of two simple krigings:

$$T^{CK}(x_0) = a\ A^K(x_0) + R^K(x_0)$$

$$\text{where } \begin{cases} A^K(x_0) = \displaystyle\sum_{\substack{\text{stations} \\ +\text{ underways} \\ \in \\ \text{neighbourhood}}} l_i^A A(x_i) \\ R^K(x_0) = \displaystyle\sum_{\substack{\text{stations} \\ \in \\ \text{neighbourhood}}} l_i^R R(x_i) \end{cases}$$

The cokriging variance is:

$$s_T^{CK}(x_0) = a^2\ s_A^K(x_0) + s_R^K(x_0)$$

The variance discrepancy above mention renders comparisons between cokriging (or its simplification) and kriging tricky. Variance rescaling is necessary, so that finally we have:

$$\sigma_T^{CK}(x_0) = \frac{\alpha^2}{\kappa^2}\sigma_A^K(x_0) + \sigma_R^K(x_0)$$

It is equivalent to standardize the variables.

Four surveys from the Barents Sea (1997, 1998, 2000, 2001) and three from the North Sea (Scotland 1999, 2002 and 2003) honour the Markov-type model assumptions. In the Irish Sea, an ordinary cokriging with no simplification was applied, as the cross structure was not proportional the acoustic structure (surveys 2000 and 2002).

- **Kriging with external drift**

In this model, the catch estimation is forced to follow spatially the shape given by the acoustic. The model with external drift is in fact quite similar to the previous model, except that the α parameter can change from point to point in the map.

In this model, the estimation is made in two steps:

1. the ordinary kriging of the acoustic underways:

$$A^K(x_0) = \sum_{\substack{stations \\ +\ underways \\ \in \\ neighbourhood}} \lambda_i^A A(x_i)$$

2. the kriging of the catch with the acoustic estimation with external drift:

$$T^K(x_0) = \sum_{\substack{stations \\ \in \\ neighbourhood}} \lambda_i^T T(x_i) \quad \text{with } E\left[T^K(x_0)\right] = \alpha + \beta A^K(x_0)$$

In practice it is similar to an ordinary kriging with just one additional constraint on the weights λ_i^T.

As the optimisation problem has more constraints, the estimation variance is necessarily larger than for an ordinary kriging. The comparison of the estimation variance is then not always the good way to find the best model. A model can have intrinsically a larger error variance but also have more suitable other kind of features.

Models comparison

Classical statistical tests are difficult to compute in geostatistics because the distributions of the variable (normal, lognormal, gamma…) are often specified and the autocorrelation between the data are generally high, at least for short distances (Chiles and Delphiner 1999).
Here we describe some classical tools to compare models outputs in such circumstances. If the data distribution is specified, more criteria like confidence intervals or statistical tests can be taken into account.

A model has to be as robust as possible to different configuration of data. Often this property is tested by bootstrapping. The parameters are calculated independently on few subsamples and the values obtained are then compared. If the values of the parameters change very little from a subsample to an other, the model is said robust. Because of the correlation between the data, cut out by subsample is difficult in Geostatistics. Removing data can slant the model.
Nevertheless, several surveys are available for different years and different area. We have considered that a robust model is a model which can be applied, i-e the assumption required have been verified, for most of the surveys. The parameters change obviously a lot from a survey to an other, because of the radical differences between the areas. We have then chosen models (the cokriging or the Markov-type model and the kriging with external drift) which are relevant for a lot of surveys.

In the following four sections we are using Barents sea output to illustrate the method. A summary of all results will be given after all.

- ### Comparison of estimated maps

First, a natural way is to compare the pattern of the estimated maps from each model. The comparison is more visual than quantitative. The pattern can be described, for example, by the regularity of the isolines for different levels. The figure 2 shows an example for the Norwegian survey 2001. The isolines of the cokriging estimation have more details than the kriging map. The additional acoustic information makes the estimation less smooth. The maximum and the minimum estimated by kriging are also less contrasted than with cokriging.

Nevertheless these added details make not necessarily a more accurate model display if the related estimation variance is high. Here, the map seems to be more realistic because less smoothed, this improvement have to be linked with a variance decrease to conclude.

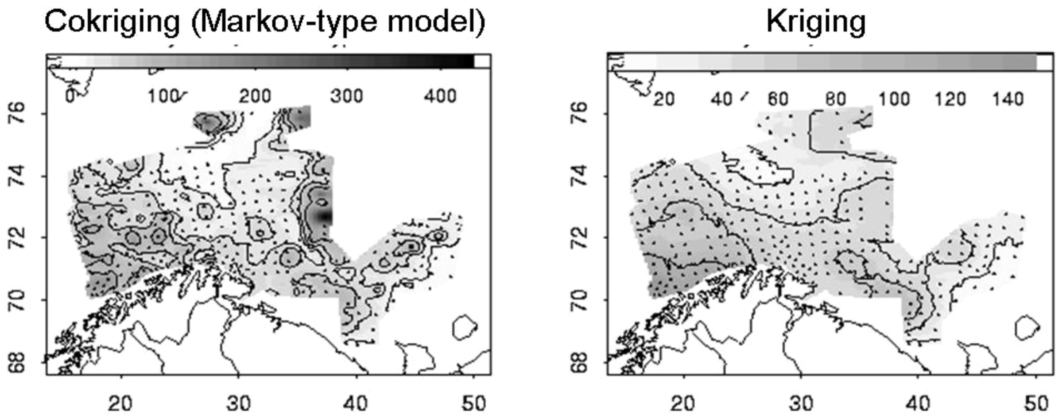

Figure 3. Estimation maps obtained by the Markov-type model (left) and a simple model using only the catch information available on station in a compatible model (right). The map on the left hand side is very more detailed. To compare the models, the grey scales are identical.

- **Local variance**

The local variance of kriging or cokriging indicates the spatial evolution of the variance model. As kriging and cokriging are exact interpolation, the variance is necessarily zero at samples points. The evolution of the variance away from the samples locations inform about the accuracy of the model. The kriging variance can be deduced from the model of variogram (or covariance) chosen and the points locations.

The variance of cokriging is necessarily below the variance of kriging, with consistent models. If the additional variable brings no accuracy, their weights will be zero and the cokriging variance will be then equal to the kriging variance. In the same way, the kriging with external drift variance is always greater than a kriging variance, because of the constraint of shape imposed to the expectation.

The comparison of the variance maps allows quantifying the effect of the use of the use an auxiliary variable. The kriging variance depends of the geometry between the points locations. Here, the auxiliary variable being more densely sampled, the variance of the combining models is systematically lower.

It is interesting to see here than even if the variance of cokriging is globally lower than the kriging's one, the variance tends locally to increase more quickly between the transects (cf figure 4).

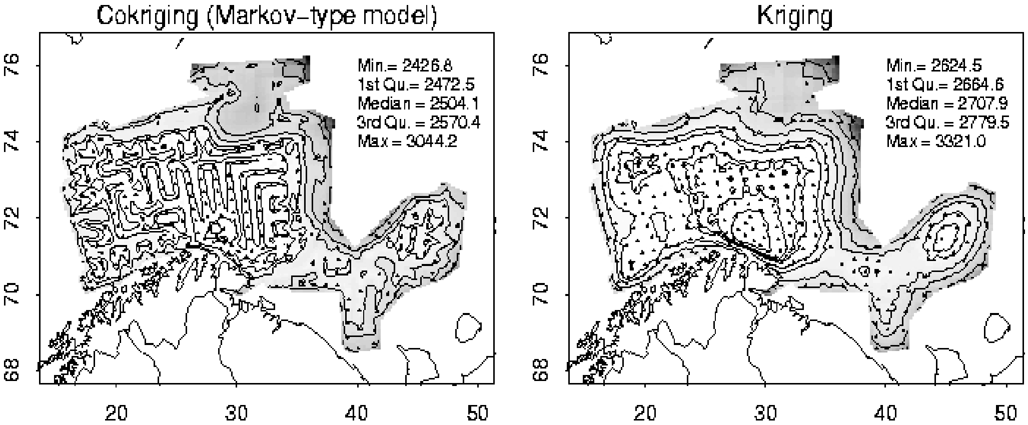

Figure 4. Estimation of the error variance obtained by the Markov-type model (left) and a simple model using only the catch information available on station in a compatible model (right).

▪ Global estimation variance

The global estimation variance s_E^2 quantifies the variance of the error performed when estimating the mean density over a given area by the arithmetic mean m_E (Rivoirard and Al. 2000)

The global estimation variance is given by the covariance function by computing three terms depending only of the geometry of the estimation pattern: the shape of the sampled field, the relative locations of the samples and finally the position of the samples in the field (Petitgas, 1991).
For a given spatial structure, the global estimation variance the efficiency of a sampling scheme. We will use it to estimate the improvement brought by the acoustic.

If m_E is an estimation of the mean, and σ_E the estimation variance, the coefficient of variation (CV) gives a measure of the relative uncertainty of the mean provided by the model (Rivoirard 2000). The CVs obtained with different models, i-e under different hypotheses, can be used to compare various models providing different estimates.
Assuming independence between samples, we get:

$\mathrm{CV}_{\mathrm{iid}} = \frac{s_E}{m_E} = \frac{s}{m\sqrt{n}}$ with σ the standard deviation of the data, m the average, for n samples independent and identically distributed.

In case of kriging or cokriging we get: $\mathrm{CV}_{\mathrm{geo}} = \frac{s_E}{m_E}$.

When data are correlated, the information is redundant. It is like if fewer data were available. The comparison between CV_{iid} and CV_{geo} allows to appraise if this loss of information is balanced by the gain of accuracy in the estimation when using explicitly the spatial structures.

- **Cross validation and analysis of the kriging errors**

Cross validation (also called leave-one-out- method) is a common approach to compare different models.
Each sample value is removed from the data set, and an estimation at that location is assessed using the n-1 other values. The difference between the data value and the estimated value $(T(x_i) - T^*(x_i))$ represents the kriging error and gives an indication of how the model is able to re-estimate the data. The estimation provides also an kriging variance: $s_K^2(x_i)$. We can then calculate the standardized errors: $(T(x_i) - T^*(x_i)) / s_K^2(x_i)$.

In a heterotopic case, the cross validation allows to verify the estimated values only for the locations where all the variables are available, here on stations where both acoustic and catch are collected.

An analysis of the kriging errors have then been inspected, as the spatial distribution (fig. 5), the histogram (fig. 6), the scatterplot between the estimation and the true values (fig. 8), or between the estimation and the errors (fig. 9). This type of analysis helps to detect erroneous data or some phenomena of discontinuity or a lack of stationarity which have to be taken into account.
Some dependence between the errors and the values or between the errors and the estimations can then be detected. The map and the histogram of the standardized errors allow indicating some zones where points are poorly estimated. The inevitable scatterplot between the estimated and the real values shows the adequacy of the assessment and an eventual bias (fig. 7).
The variogram of the errors have even been calculated to detect spatial dependence (fig.10). The results of two models can be confronted by a cross plot (fig.11).
All theses methods allows also comparing properties of estimators performed under different models with different assumptions.

The average of the ratio between the cross validation error and the kriging standard deviation provides a good quantity to compare models with taking into account the error predicted by the kriging.

$$m.s.s.e = \frac{1}{n}\sum_{i=1}^{n}\frac{\left(T(x_i)-T^*(x_i)\right)^2}{s_K^2(x_i)} \simeq 1$$

Closer to one it is, better is the predictive capacity of the model. Other indices like the error average or the root-mean-squared error appraise the model quality.

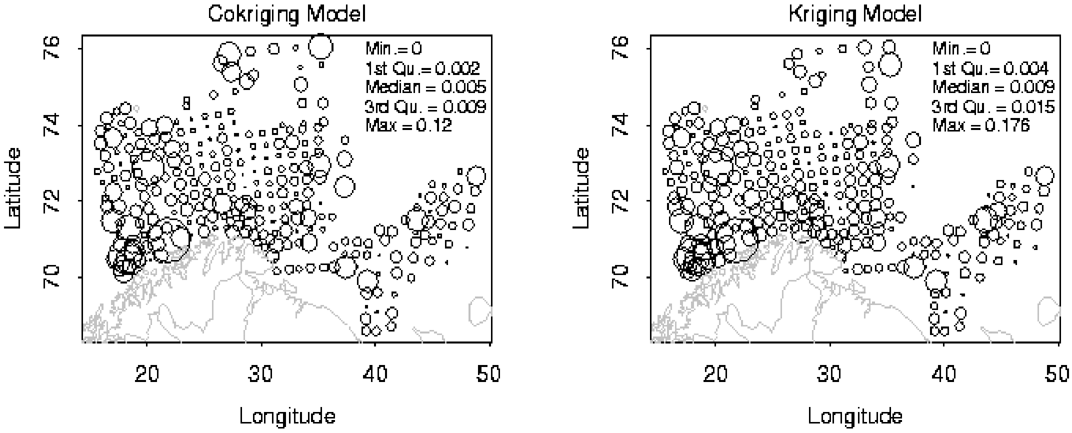

Figure 5. Proportional representation of the absolute value of the standardized errors for two models. Left: the cokriging (Markov-type model) combined acoustic and catch, and right: the only catch model (kriging).

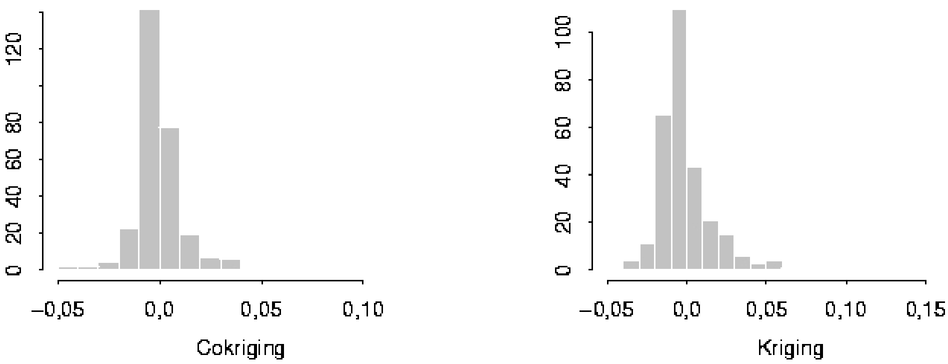

Figure 6. Histograms of the standardized errors for the two models with acoustic (left) and without (right), for the survey Norway 2001.

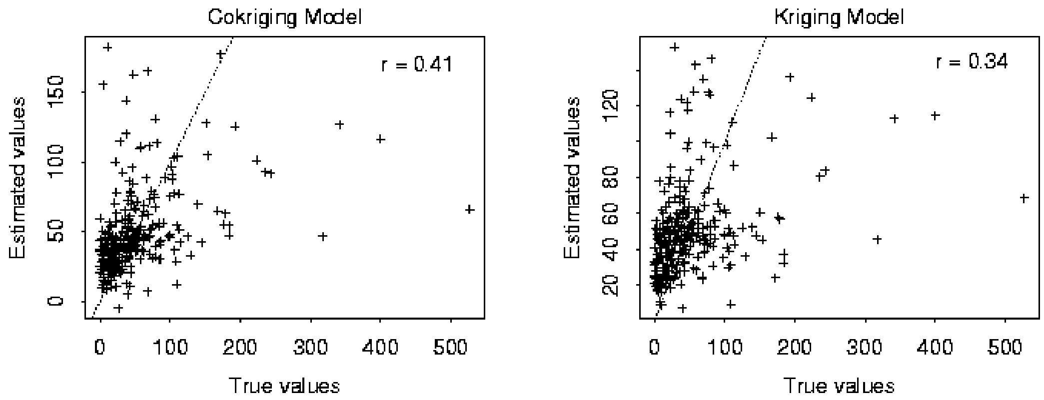

Figure 7. Scatterplot between the true values and the estimated values by cross validation. Left: combined model and right: only-catch model, Norway 2001.

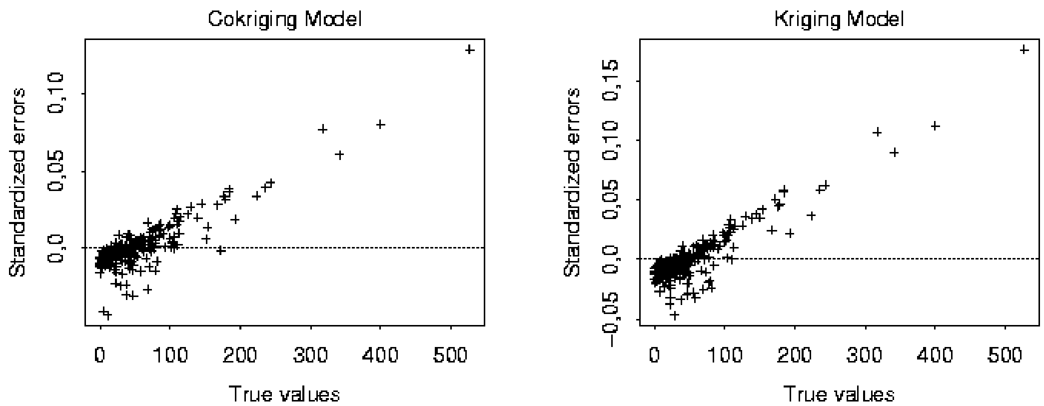

Figure 8 Scatterplot between the true values and the standardized errors obtained by cross validation. Left: combined model, right: only catch model, Norway 2001.

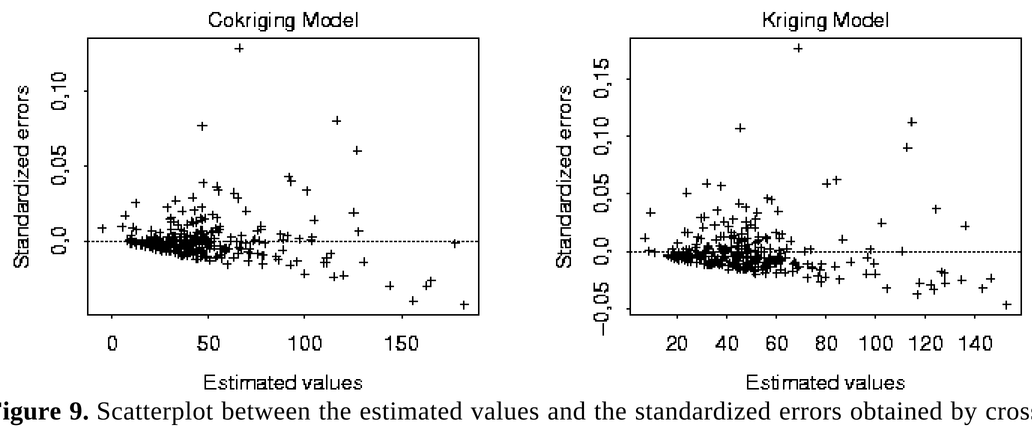

Figure 9. Scatterplot between the estimated values and the standardized errors obtained by cross validation. Left: combined model, right: only catch model, Norway 2001.

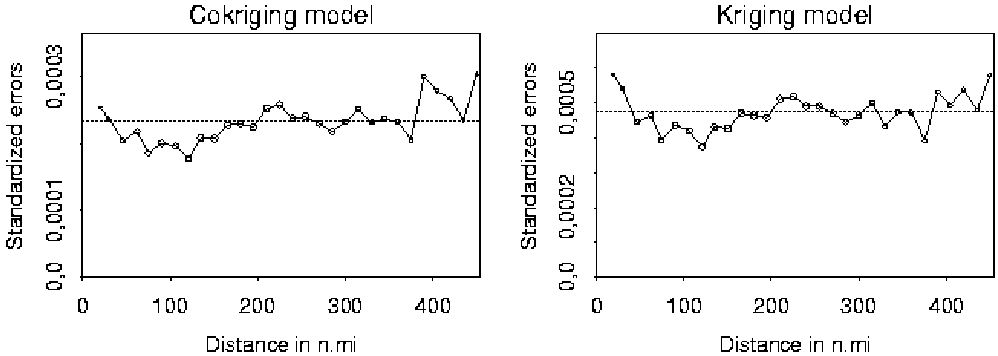

Figure 10. Variogram of the standardized errors obtained by cross validation. Left: combined model, right: only catch model, Norway 2001.

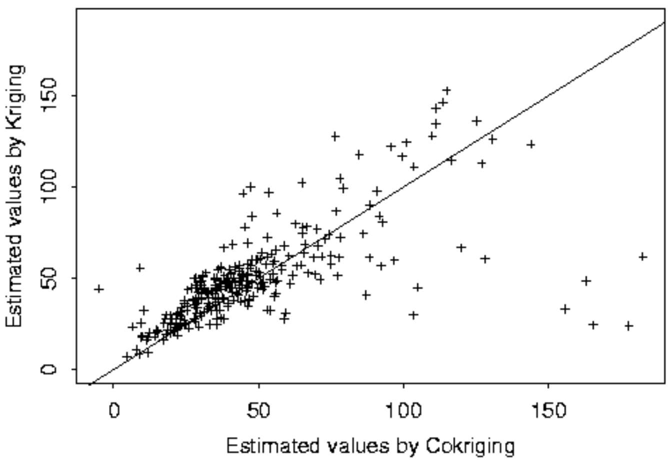

Figure 11. Crossplot between the estimated values by cokriging (combined model) and by kriging (only catch model), Norway 2001.

General Results

The addition of the acoustic provides an improvement of the details in the estimation maps for all the surveys tested. The combined model are always less smooth than the model using only the catch.

The local and global variances are often less with the combined model. It indicates that the gain of details is joined with an increase of precision.
The cross validation errors seem to have similar behaviour of the different models. The acoustic provides less flat estimations in particularly for the Norwegian and Irish surveys.

As expected, cokriging gives most of the time better results than the kriging with external drift (Table 2), except for Scottish data. The disappointed results obtained in Scotland are probably due to the sensitivity of the model to the outliers which are more numerous when more data are used (like for the combined model).

Most of the quality indices indicate the combined model (with cokriging) superior to the model using only the catch variable for Norwegian data (Table 2). In Ireland, even if the cross validation indicates similar results between the models, the global variance and the CV are less for cokriging than for kriging suggesting that the acoustic brings precision.

These differences in the models outputs between the countries can be attributable to the diverse surveys designs. Because of the present of autocorrelation, geostatistical tools are more efficient in a regular design than a random stratified design context.

The dissimilar depth and trawl geometry between the three sites can also have an important impact on the correlation acoustic-catch, and then on the combined model effectiveness. Deeper is the water column, larger is the acoustic cone at the bottom and farer is the trawl from the vessel. The difference between the size of the trawl opening and the acoustic cone diameter can explain the divergences observed. The species, the environment and the stock abundance can also affect the acoustic-catch link.

Finally, the number of samples collected on-stations and between stations is perhaps the very reason why the combined acoustic-catch model works better for the Norwegian surveys. As a matter of fact, models need a lot of data, especially when data distributions are so skew.

Acknowledgement

The authors thank Vidar Hjellvik of the Institute of Marine Research of Bergen, Doug Beare and Tony Greig of the Marine Laboratory of Aberdeen, Mike Amstrong (in this time) at the Queen’s University of Belfast, and all the others CATEFA partners (CEFAS, IFREMER).

They carry out all the surveys used in this study, formatting the databases and helping our understanding of the Barents Sea, the North Sea and the Irish Sea features. The comments about geostatistical models discussed during the working groups of CATEFA have been very helpful for the achievement of this work.

Table 2. Results of some performance indices for each models tested: the kriging using only the catch data, the cokriging (of Markov-type model) and the kriging with external drift which combined acoustic and trawl. s_E^2 is the global variance estimation, CV is the geostatistical coefficient of variation, $r(Z^*, Z)$ represents the correlation between the estimated values (by cross validation) and the real values, RMS is the root-mean-squared errors of cross-validation and then m.s.s.e is explain above.

Country	Year	Model	s_E^2	CV_{iid}	CV_{geo}	$r(Z^*, Z)$	RMS	m.s.s.e
Norway	1997	Kriging	27.14	0.128	0.074	0.16	120.9	1.86
		Cokriging	15.48		0.062	**0.51**	**101.7**	**1.76**
		Kriging with ED				0.06	141.2	1.86
	1998	Kriging	5.63	0.084	0.058	0.09	47.9	1.71
		Cokriging	3.47		0.046	**0.29**	**44.7**	1.81
		Kriging with ED				0.14	47.9	**1.36**
	2000	Kriging	11.61	0.424	0.629	0.06	40.1	1.82
		Cokriging	0.48		0.118	**0.38**	**36.3**	**0.73**
		Kriging with ED				0.29	38.1	0.53
	2001	Kriging	8.82	0.07	0.059	0.34	56.6	1.23
		Cokriging	7.63		0.107	**0.41**	**54.9**	0.84
		Kriging with ED				0.38	60.7	**1.04**
Scotland	1999	Kriging	0.512	0.515	0.515	0.061	4.7	1.03
		Cokriging	2.460		15.48	0.041	10.6	0.13
		Kriging with ED				**-0.34**	**4.8**	**1.03**
	2002	Kriging	13.99	0.159	0.125	0.247	37.65	0.93
		Cokriging	21.90		0.093	0.228	45.57	0.45
		Kriging with ED				**0.504**	**31.85**	**0.979**
	2003	Kriging	5.46	0.165	0.118	0.595	17.86	1.15
		Cokriging	4.31		0.086	0.615	17.11	**1.03**
		Kriging with ED				**0.627**	**16.97**	0.048
Ireland	2000	Kriging	2.23	1.48	0.112	0.594	**15.02**	**1.16**
		Cokriging	2.04		0.107	**0.614**	15.57	1.43
		Kriging with ED				0.514	33.49	1.56
	2002	Kriging	3.99	1.85	0.182	0.528	17.97	1.91
		Cokriging	2.34		0.139	0.540	**17.70**	**1.80**
		Kriging with ED				**0.543**	18.18	1.95

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